

**BACHELOR IN COMPUTER
APPLICATIONS****Term-End Examination****December, 2008****CS-601 : DIFFERENTIAL AND INTEGRAL
CALCULUS WITH APPLICATIONS**

Time : 2 hours

Maximum Marks : 60

Note : Question number 1 is **compulsory**. Answer any **three** questions from the rest. Use of calculator is allowed.

1. (a) For real numbers x and y , tell for each of the following, whether it is True or False :

(i) $|x + y|$ always equals $|x| + |y|$

(ii) $|x \cdot y|$ always equals $|x| \cdot |y|$

(iii) $|x - y|$ is greater than or equal to $||x| - |y||$,

where $|x|$ denotes absolute value of x .

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(b) Fill in the blanks :

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(i) By Trichotomy Law in real numbers, if a and b are two real numbers then one and only one of the following holds

$$a > b, \quad a = b, \quad \dots\dots\dots$$

(ii) By commutative property of ' \cdot ' in \mathbb{R} , we get

$$a \cdot b = \dots\dots\dots$$

for all real numbers a, b .

(iii) By associative property of '+' in \mathbb{R} , we get

$$(x + y) + z = \dots\dots\dots$$

for all real numbers x, y and z .

(c) In each of the following, if $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ is a function and is defined as

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(i) $f(x) = 5x + 6$, then tell whether f is 1 - 1 or not and why ?

(ii) $f(x) = 6x^2$, then tell whether f is onto or not and why ?

(iii) $f(x) = 3/x^2$, then tell whether f is 1 - 1 or not and why ?

(d) Given : $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ are two functions such that $f(x) = 5x^2 - 3$,
 $g(x) = 8x - 2$, then find fog and gof.

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(e) Find dy/dx for each of the following :

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(i) $y = 3 \cos x - 7$

(ii) $y = 8 + 11x$

(iii) $y = 6x^2$

(f) Evaluate each of the following : 3

(i) $\int (4x^2 + 7) dx$

(ii) $\int (3 \sin x) dx$

(iii) $\int 17 dx$

(g) Evaluate each of the following : 3

(i) $\int_1^2 (2x + 3) dx$

(ii) $\int_3^5 e^{3x} dx$

(iii) $\int_{\pi/4}^{\pi/2} 2 \cos x dx$

(h) Which of the following functions is monotonically increasing or monotonically decreasing or neither, on the given interval ? 4

(i) $3x^2 - 6$ on the interval $[0, 3]$

(ii) $\sin x$ on the interval $[0, \pi/2]$

(k) Find the area under the part of the curve $y = 2x + 3$ that lies between $x = 0$ and $x = 2$ and is also bounded by $y = 0$. 4

2. (a) State the following properties/laws of real numbers : 4
- (i) Commutative property of addition
 - (ii) Associative property of multiplication
 - (iii) Archimedean property
 - (iv) Distributivity of multiplication over addition
- (b) Draw a graph of each of the following functions : 4
- (i) $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = 7$ for all x in \mathbb{R}
 - (ii) $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = 2x^2 + 3$ for all x in \mathbb{R} .
- (c) Define the following concepts with an example for each : 2
- (i) Periodic function
 - (ii) Monotone function
3. (a) Evaluate the following : 6
- (i) $\int (18x^6 + 5x^4 + 2) dx$
 - (ii) $\int \frac{13}{24(1+x^2)} dx$
 - (iii) $\int \frac{23}{48(x\sqrt{x^2-1})} dx$

(b) Evaluate the following : 4

(i) $\int_0^{\pi/2} \frac{114}{183} \sin 8x \cos 6x \, dx$

(ii) $\int_3^7 \frac{x^3 + 2x}{x^2 - x - 2} \, dx$

4. (a) Show that the function $f(x) = 11x^4 + 13$ is monotonically increasing over \mathbb{R} , the set of real numbers. 4

(b) Show that $3e^x + 3e^{-x} > 6$ for all x in \mathbb{R} . 2

(c) Find the area under the curve $y = 12 \cos x$ bounded by the line $x = 0$, $x = \pi$ and $y = 0$. 4

5. (a) Give Taylor's series expansion for the following : 4

(i) e^x

(ii) $\ln(1 + x)$

(b) Write Maclaurin's series expansion for each of the following : 4

(i) $\cos x$

(ii) $(x - 2)^2 + 1$

(c) Find/Give dy/dx for each of the following : 2

(i) $y = 3 \cos^{-1} x$

(ii) $y = 7 \operatorname{cosec}^{-1} x$

