

# BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

June, 2007

## CS-601 (S) : DIFFERENTIAL AND INTEGRAL CALCULUS WITH APPLICATIONS

Time : 2 hours

Maximum Marks : 60

**Note :** Question number 1 is **compulsory**. Answer any **three** questions from the rest.

1. (a) State whether the following statements are true or false. Give reasons for your answers. 5×2=10
- (i)  $f(x) = 3 - |x - 2|$  is differentiable in  $[1, 5]$ .
- (ii)  $\frac{d}{dx} \left[ \int_0^{2x} \cos t^2 dt \right] = 2 \sin 4x^2$
- (iii) If  $|x + 3| \leq 1$ , then  $x \in [-4, -2]$ .
- (iv) A continuous function is differentiable.
- (v)  $x^2 - x + 2$  is an increasing function on  $[0, 2]$ .

- (b) Examine whether the following functions satisfy the conditions for Rolle's theorem on the given intervals.

$$2\frac{1}{2} \times 2 = 5$$

(i)  $f(x) = x \ln x + x - 2$  on  $[1, 3]$

(ii)  $f(x) = x^2 - 3x + 2$  on  $[1, 2]$

(c) Evaluate :  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 - 5x + 6}$  3

(d) Evaluate :  $\int_0^{\pi/2} (\sin x + \cos x)^2 dx$  3

- (e) The sum of two numbers is 20. Find these numbers if the product of the square of the first and cube of the second is maximum. 3

- (f) A curve is drawn to pass through the points, given by

x	0	1	2	3	4	5	6
y	0.14	0.16	0.17	0.19	0.20	0.21	0.23

Estimate the area bounded by the given curve and x axis, between the ordinates  $x = 0$  and  $x = 6$  by using trapezoidal rule. 4

- (g) If  $y = 2 \sin x + 3 \cos x + 5$ , prove that  $y'' + y = 5$ . 2

2. (a) Define the greatest integer function  $[x]$ . Draw its graph on  $[-2, 6]$ . 3

(b) If  $y = e^{m \tan^{-1} x}$ , prove that  $(1 + x^2) y_2 + (2x - m) y_1 = 0$ . 3

(c) Find the area of the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$  using integrals. 4

3. (a) Show that

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{2}{3},$$

if  $n$  is odd, and hence find  $\int_0^{\pi/2} \sin^7 x \, dx$ . 5

(b) Find the equation of the normal at  $t = \pi/4$  to the curve given by  $x = a(\cos t + t \sin t)$ ,  $y = a(\sin t - t \cos t)$ . 5

4. (a) Find the domain and range of the function

$$f(x) = \sqrt{\frac{x+1}{x+4}}. \quad 3$$

(b) Prove that the curves  $y^2 = 16x$  and  $2x^2 + y^2 = 4$  cut each other orthogonally. 3

(c) Evaluate  $\int_0^{\pi/2} \frac{\cos x \, dx}{(1 + \sin x)(2 + \sin x)}$  4

5. (a) Find the asymptotes parallel to either axis (if there are any) for the following curve : 3

$$x^2y - y - 2 = 0$$

- (b) Find the curvature of  $f(x) = x^2 + 9$  at  $(0, 9)$ . 4

- (c) Find the approximate value of  $(1.01)^{5/2}$  by using the first three terms of the Maclaurin's expansion of  $(1 + x)^{5/2}$ . 3