MCA (Revised)

Term-End Examination

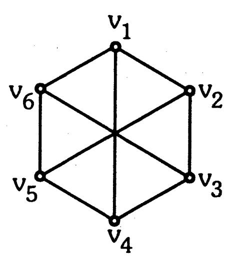
June, 2007

MCS-033 Ⓢ : ADVANCED DISCRETE MATHEMATICS

Time : 2 hours *Maximum Marks : 50*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest. Calculators are* ***not*** *allowed.*

1. (a) Consider the following graph :



(i) Find a walk of length 7 in the graph. Is it a path ?

(ii) Write the degree sequence of the graph.

(iii) Draw the complement of the graph. Is the complement connected ? 4

(b) Write the order and degree of the recurrence

$a_{n+1} = a_n + 2n(2n - 1)\, a_{n-1}, \; n \geq 2.$

Is the recurrence homogeneous ? Check that

$a_n = \dfrac{(2n)!}{2^n . n!}$ satisfies the recurrence. Also state the initial conditions that make it satisfied. 4

(c) State the sufficient conditions of Dirac and Ore for a graph to be Hamiltonian. Give an example of a graph that does not satisfy Dirac's condition, but satisfies Ore's condition. 4

(d) Find a particular solution to the recurrence

$a_{n+1} - 2a_n + a_{n-1} = 5 + 2^n, \; n \geq 1$ 5

(e) Is the graph G with degree sequence $\{2, 3, 3, 4, 4, 4, 4\}$ planar ? Justify your answer. If the graph is planar draw the graph. 3

2. (a) Use generating function to solve the recurrence relation

$a_n - 6a_{n-1} + 11a_{n-2} - 6a_{n-3} = 0, \; (n \geq 3)$

where $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$. 7

(b) Let G be a graph with

$V(G) = \{ \{1, 2\}, \{1, 7\}, \{5, 3, 7\}, \{3, 7\}, \{6, 8\}, \{6, 4\} \}$

For two sets A, B $\in$ V(G), AB $\in$ E(G) if $A \cap B \neq \phi$. Draw the graph G and find its connected components. 3

3. (a) Let a_n be the number of words (need not be meaningful) of length n that can be formed using the letters A, B and C such that any A or B has to be followed by a C.

(i) Find a_1, a_2 and a_3.

(ii) Find a recurrence relation for a_n. 6

(b) Define a regular graph. Draw 3-regular graphs on 6 vertices. Are any two 3-regular graphs on 6 vertices isomorphic ? Justify your answer. 4

4. (a) Find the number of integer solutions to $a_1 + a_2 + a_3 = n$ where $-2 \le a_1 \le 2$, $1 \le a_2 \le 5$, $a_3 \ge 2$ and $n \ge 0$ is an integer. 6

(b) Define spanning tree of a graph. Can you give an example of a graph which has a unique spanning tree upto isomorphism. Is it true that any graph has a unique spanning tree upto isomorphism ? 4

5. (a) Use an appropriate substitution to solve the recurrence :

$$x_n = \left(\sqrt{x_{n-1}} + 2\sqrt{x_{n-2}}\right)^2, \; n \ge 2, \; x_0 = 1, \; x_1 = 1$$ 5

(b) For the following graphs find a minimal colouring. 5