UNIT 9 INTERFEROMETRY

Structure

9.1 INTRODUCTION

Light is considered as an electro-magnetic wave of sinusoidal form. When two monochromatic light beams combine they undergo the phenomenon of interference. Therefore, the resultant light rays carry the characteristics of both the monochromatic light sources. The amplitude and hence the brightness of the resultant beam also becomes different from the original ones. The rays of light, when allowed to fall on screen, form alternate dark and bright fringes from maximum intensity to complete minimum as the phase of the monochromatic beams changes from 0 to 180 degree. By analyzing differences in phase, which also correspond to difference in wavelength (length), distance between the two lights sources can be calculated. This can be a tool to measure very small linear differences. Phenomenon of light used in this way has given rise to the branch of dimension metrology called Interferometry. Interferometry involves testing of flatness, surface contour, and determination of the thickness of slip gauges, etc. Interferometers are commercially available instruments for calculating small differences, constructed on the basis of principles of interferometry. We will discuss these instruments and the techniques used in them in the following sections.

Objectives

After studying this unit, you should be able to

- familiarise yourself with the principle of interference, and
- understand the techniques and working of various devices based on the phenomenon of interference.

9.2 INTERFERENCE OF LIGHT

To understand the phenomenon, let us consider two virtual sources S_1 and S_2 derived from the same primary source. Since both are equidistant from the source *S*, the secondary wavelets emerging from S_1 and S_2 are in same phase and have equal amplitude.

Let the source slit *S* be illuminated with a monochromatic light of wavelength λ. The identical light waves diverging from S_1 and S_2 at an instant *t* may be represented by

$$E = E_0 \sin \frac{2\pi t}{T} = E_0 \sin \frac{2\pi c t}{\lambda} \qquad \ldots (9.1)$$

where E_0 is the amplitude, T is the period, and c is the velocity of light.

In traveling from S_1 to P, the wave travels a path of $x_1 = S_1 P$. Hence, on reaching P, the wave that emerged looks like

$$E_1 = E_0 \sin\left\{\frac{2\pi}{\lambda}(ct - x_1)\right\} = E_0 \sin(\omega t - \phi_1) \qquad \text{. . . (9.2)}$$

where, $\omega = \frac{2\pi c}{\lambda} = \frac{2\pi}{T}$ is the angular frequency and $\phi_1 = \frac{2\pi x_1}{\lambda}$ the phase of the wave.

Similarly, on reaching P, the wave emerged from S_2 is given by

$$E_2 = E_0 \sin\left\{\frac{2\pi}{\lambda}(ct - x_2)\right\} = E_0 \sin(\omega t - \phi_2)$$

where $x_2 = S_2$ P and $\phi_2 = \frac{2\pi x_2}{\lambda}$ is the phase of the wave.

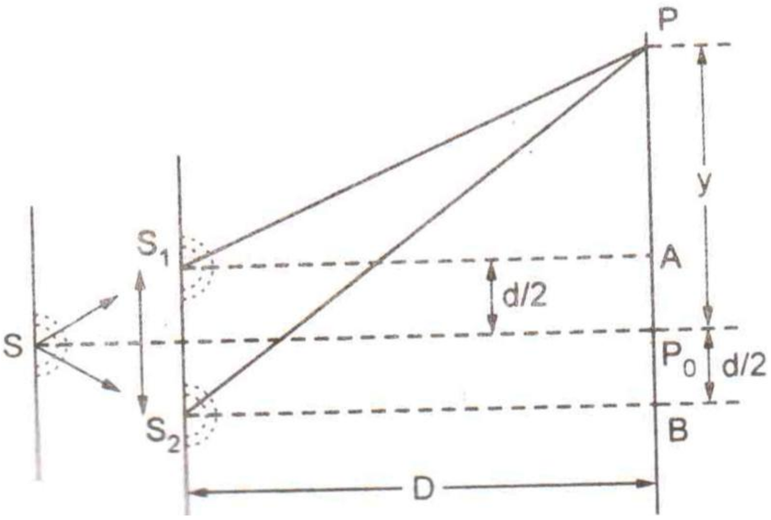


Figure 9.1

The phase difference between the two sources is

$$\phi = \phi_2 - \phi_1 = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi\Delta}{\lambda}$$

where the path difference $\Delta = x_2 - x_1 = S_2 P - S_1 P$

Now assuming that $y << D$ and $d << D$, i.e. the slits are very close together, we can consider electro-magnetic fields E_1 and E_2 at P very nearly in the same direction. In this condition, superposition of E_1 and E_2 is simply the ordinary sum of E_1 and E_2.

The resultant wave is given by

$$E = E_1 + E_2$$

$$= E_0 \sin(\omega t - \phi_1) + E_0 \sin(\omega t - \phi_2)$$

$$= E_0 [(\cos\phi_1 + \cos\phi_2)\sin\omega t - (\sin\phi_1 + \sin\phi_2)\cos\omega t] \qquad \text{. . . (9.3)}$$

Let, $$E_0 (\cos\phi_1 + \cos\phi_2) = R\cos\theta \qquad \text{. . . (9.4)}$$

$$E_0 (\sin\phi_1 + \sin\phi_2) = R\sin\theta \qquad \text{. . . (9.5)}$$

Substituting Eqs. (9.4) and (9.5) in Eq. (9.3), we have

$$E = R\sin(\omega t - \phi) \qquad \text{. . . (9.6)}$$

Thus, the resultant amplitude of the new ray is R and phase θ.

The intensity depends on the square of the resultant amplitude. Squaring and adding Eqs. (9.4) and (9.5)

$$R^2 = 2E_0^2\,[1 + \cos(\phi_2 - \phi_1)]$$

$$= 2E_0^2\,(1 + \cos\phi)$$

As the value of cos ϕ varies between – 1 and 1, intensity also varies from 0 to $4E_0{}^2$. The resultant intensity I is maximum if R^2 is maximum. R^2 will be maximum if

$$\cos\phi = +1 \text{ or } \phi = 0, 2\pi, 4\pi, \ldots$$

Hence, $$\frac{2\pi\Delta}{\lambda} = 0, 2\pi, 4\pi, \ldots$$

Hence, $$\Delta = 0, \lambda, 2\lambda, \ldots$$

The maximum value of I is

$$I_{\max} = K R^2 = 4K E_0^2$$

where K is the proportionality constant. The interference in this condition is said to be constructive interference.

The resultant intensity is minimum if $\cos\phi = -1$ or $\phi = 0, 3\pi, 5\pi, \ldots$

i.e. $$\frac{2\pi\Delta}{\lambda} = \pi, 3\pi, 5\pi, \ldots$$

which means $$\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

The minimum value is, $I_{\min} = 0$. The interference in this condition is said to be destructive interference.

Thus the condition for maximum and minimum of intensity is

Maxima at $\Delta = S_2P - S_1P = m\lambda,\ m = 0, 1, 2, 3, \ldots$

Minima at $\Delta = S_2P - S_1P = \left(m + \frac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots$

This maxima and minima constitute the bright and dark fringes.

From Figure 9.1, we can easily calculate position of the bright and dark fringes.

$$(S_2P)^2 = (S_2B)^2 + (BP)^2$$

$$= D^2 + \left(y + \frac{d}{2}\right)^2$$

$$S_2P = D\left(1 + \frac{\left(y + \frac{d}{2}\right)^2}{D^2}\right)^{\frac{1}{2}}$$

Since we have assumed that $y << D$, $d << D$, we can expand the term binomially,

Hence, $$S_2P = D + \frac{1}{2D}\left(y + \frac{d}{2}\right)^2$$

Similarly, $$S_1 P = D + \frac{1}{2D}\left(y - \frac{d}{2}\right)^2$$

Therefore, $$\Delta = S_2 P - S_1 P = \frac{1}{2D} \times 4y \times \frac{d}{2} = \frac{yd}{D}$$

Now for bright fringes :

$$\Delta = m\lambda$$

Hence, $$y_m = \frac{d}{D} = m\lambda$$

which gives, $$y_m = \frac{myD}{d} \qquad \text{. . . (9.7)}$$

In the above relation, value of y_m gives position of m^{th} bright fringe. For the central fringe at P_0, $m = 0$ and $y_0 = 0$ and the next bright fringe will be at $y_1 = \frac{D\lambda}{d}$ and so on. For the $(m-1)^{\text{th}}$ bright fringe, we have

$$y_{m-1} = \frac{(m-1)\,D\lambda}{d}, \text{ where } m = 1, 2, 3, \ldots$$

∴ The fringe width

$$y_m - y_{m-1} = \frac{D\lambda}{d} \qquad \text{. . . (9.8)}$$

Similarly, the dark fringes satisfy the relation

$$y_{m'} = \left(m - \frac{1}{2}\right)\frac{D\lambda}{d} \qquad \text{. . . (9.9)}$$

The first dark fringe is at ($m = 1$)

$$y_1' = \frac{1}{2}\frac{D\lambda}{d}$$

The second dark fringe is at ($m = 2$)

$$y_2' = \frac{3}{2}\frac{D\lambda}{d}$$

and so on.

The separation between two consecutive dark fringes is $\frac{D\lambda}{d}$.

Thus, the fringes are equally spaced. There is a dark fringe between two bright fringes Fringe width is directly proportional to D and inversely proportional to d. Intensity of all the bright fringes is the same and the intensity of dark fringes is zero.

Distribution of intensity of the fringes follow the following equation

$$I = 2K E_0^2 \,(1 + \cos\phi) = I_0 \cos^2\left(\frac{\phi}{2}\right) \qquad \text{. . . (9.10)}$$

From the equation it is seen that the maximum intensity is $I_0 = 4K E_0^2$, where K is the proportionality constant. In terms of y, d, D, and λ, Eq. (9.5) can be written as

$$I = I_0 \cos^2\left(\frac{\pi y D}{\lambda d}\right) \qquad \text{. . . (9.11)}$$

SAQ 1

(a) How is phase of light beam related to the fringes formed due to interference?

(b) Explain the formation of white and dark fringes due to interference of light.

(c) What are the differences between constructive and destructive interference?

9.3 LIGHT SOURCES FOR INTERFEROMETRY

As mentioned earlier, it should be noted that the light source used in interferometry should be a monochromatic in nature and their amplitude should be same or at least comparable with each other. Otherwise, interference will not be visible. Ordinary light contains a mixture of light with a number of wavelengths. Hence, it cannot be considered as a source light for interferometry.

A variety of light sources are available for interferometry work but the selection of proper sources for any application depends on the requirements or results to be obtained by the interferometer, cost and convenience. Characteristics for various light sources are summarized below.

Mercury

It is less expensive source having high intensity, and green line can be easily isolated with filters. Since natural mercury contains several isotopes, each isotope emits light whose wavelength is slightly different from other. As a result, natural mercury light source radiates a mixture of slightly different but close to each other wavelengths that can be treated as monochromatic light.

Mercury 198

It is pure isotope produced by neutron bombardment of gold. It is considered to be one of the best sources of very sharply defined wavelengths, and fringes are visible with path difference upto 500 mm. Light is emitted when mercury 198 is excited by microwave produced electric field. It is the international secondary standard of wavelength.

Cadmium

It is the only natural material producing a spectral line (red) almost completely monochromatic. It can be conveniently used up to a path difference of about 200 mm. Cadmium 114 is the official secondary international standard of length.

Krypton

It is used in some instruments for its advantage of being easily excited. It is not as monochromatic as krypton-86 because krypton is a mixtures of isotopes. It can be used up to path difference of 375 mm.

Krypton 86

Krypton-86 lamp produces spectral lines of different wavelengths and, therefore, an elaborate monochromator is required to separate them. Further, its excitation takes place at very low temperature. So this lamp is used only in standardizing laboratories. It enables the fringes to be observed with maximum path differences nearing up to 800 mm.

Thallium

As 95% of its light is emitted at one green wavelength, it can be used over a reasonable path difference without the use of filter.

Sodium

It is used only in applications where interference path difference does not exceed a few hundred wavelengths. Usually yellow sodium light is used which contains two separate but closely spaced lines of equal intensities.

Helium

Orange line of helium is used where path difference is not large.

Gas Lasers

Lasers are highly monochromatic light that enable the interference fringes to be observed with enormous path differences, up to 100 million wavelengths.

SAQ 2

(a) Why ordinary light cannot be used as a source of light in interferometry?

(b) What are the various light sources commonly used in interferometry?

9.4 INTERFEROMETRY APPLIED TO FLATNESS TESTING

Flatness is defined as the geometrical concept of a perfect plane. It is an important function in construction of many technical components where accuracy is a required criterion. For example, controlled flatness is required to provide full contact with a mating part. Flatness is a precondition for the parallelism of nominally flat surface. It is a reliable boundary plan for linear dimension. It also provides locating planes for dependable mounting or assembly of manufactured parts.

Considering the functional role of flatness, the measurement of that condition on the operating surface of manufactured parts is often an important operation of the dimensional inspection process. There are various methods and instruments available for measuring the flatness of a plane. The choice of the best-suited method is governed by various factors, such as size and shape of the part, the area to be inspected, its accessibility and intersection with other surfaces and the desired degrees of measuring accuracy.

Interferometry is one of the precise methods for calculation of flatness. In this method monochromatic light is allowed to fall on an optical flat which in turn is placed on the surface at small angle whose flatness is to be calculated. Optical flats are transparent, flat, circular section of small thickness usually made of clear fused quartz or pyrex. The optical flat is placed on the surface to be inspected in such a way as to create interference bands observable under monochromatic light. The resulting band pattern permits the object's flatness conditions to be evaluated. The principle is explained below.

When optical flat is placed on the surface plate, it makes a very small angle (say, θ) to it. Monochromatic light of known wavelength is allowed to fall on it. This is shown in Figure 9.2. Part of the beam passes through the glass, strikes the flat reflecting surface, and reflected back. The other part of the beam is reflected back to air at the lower surface of the optical flat. These two beams again recombine and undergo the phenomenon of interference since they are of same wavelength. This results in formation of interference fringes as explained in Section 9.2. The position of the fringe width will depend on the separation between the surface and the lower surface of the optical flat. Considering that the angle between the surface and the optical flat is very less, the extra distance that the reflected light from the inspected surface travels is 2*d* more than the reflected light from the optical surface, where d is the separation between optical flat and the reflecting surface.

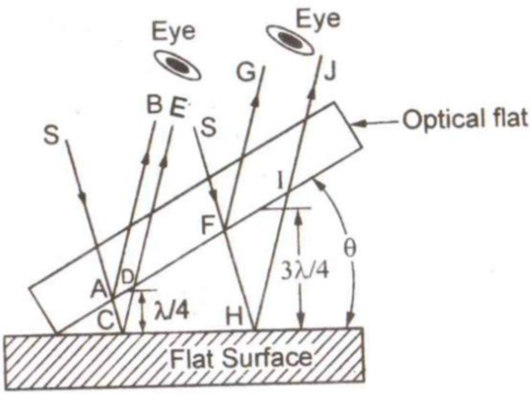


Figure 9.2

For constructive interference, the path difference between the rays should be even multiple of $\frac{\lambda}{2}$, where λ is the wavelength of light.

i.e. $$2d = 2n\frac{\lambda}{2} \text{ where } n = 1, 2, 3, \ldots$$

i.e. $$d = n\frac{\lambda}{2}$$

Thus, where there is a separation of integral multiple of $\frac{\lambda}{2}$, bright fringes will result.

On the other hand, for destructive interference, the path difference between the rays should be odd multiple of $\frac{\lambda}{2}$.

i.e. $$2d = (2n-1)\frac{\lambda}{2}$$

i.e. $$d = (2n-1)\frac{\lambda}{4}$$

Thus, the separation giving odd multiples of $\frac{\lambda}{4}$ will result dark fringes.

By studying the pattern of the fringe formed nature of the reflecting surface can be studied. A perfectly flat surface will result exactly parallel fringes without any distortion. Deviation from the perfect flatness causes digression in parallelism of the fringes proportional to flatness error.

SAQ 3

Calculate the difference in height between a slip gauge pile and a block as shown in Figure 9.5. The number of fringes over the distance *L* is 18. The source of light is cadmium having wavelength of 9.644 μm.

9.5 INTERFEROMETERS

Interferometers are optical instruments used for measuring flatness and determining minute differences in length by direct reference to the wavelength of light. Basically, an interferometer is constructed using the same principle as of an optical flat. The disadvantages of optical flats are overcome here by some refined arrangements. The fringes formed can be oriented to the best advantage in interferometers. Also there is an arrangement to view the fringes directly from top, thus avoiding any distortion due to incorrect viewing. This makes their uses easier and faster than the optical flats.

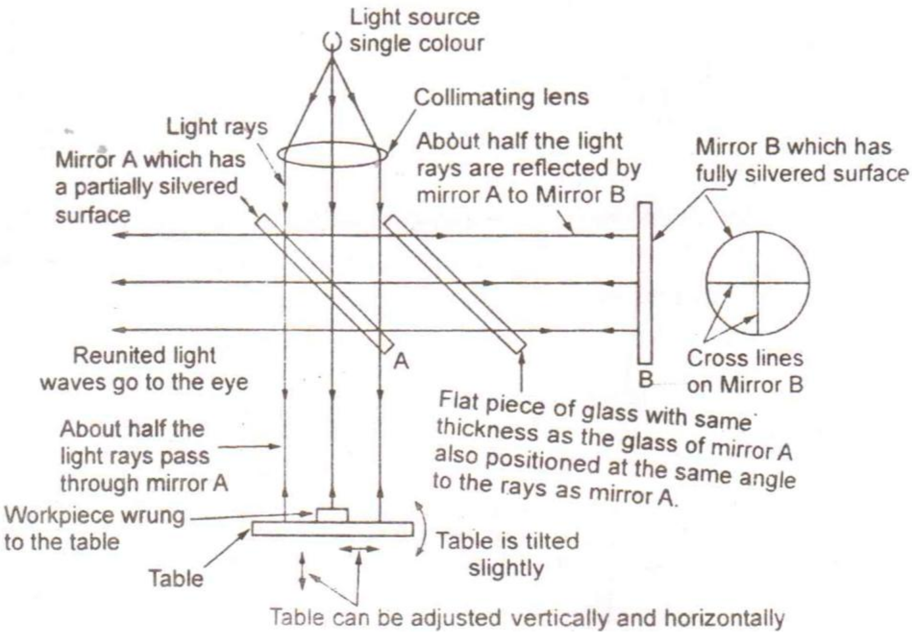


Figure 9.3 : Interferometer

There are a number of interferometers available. But most of them are designed employing slightly different methods to accomplish the same result. The arrangement of a typical interferometer is shown in Figure 9.3. The light sources from a single colour light source are collimated into parallel by a lens. When these rays reach the partially silvered surface of mirror A, about half of the light is reflected towards mirror B, and the other half passes through the silvered surface towards the workpiece and table surface. Thus, the light rays are divided and directed along two different paths. These divided light rays fall on the surface and the mirror B and are then reflected back to mirror A. Since they are of same wavelength and emerge from the same source, they will show the phenomenon of interference. Some light from the mirror B passes through the partially silvered surface towards the eye, and some light from the workpiece and table surfaces also are reflected towards the eye. If the path difference of these two rays are even multiple of the half of the wavelength of the light source, they undergo constructive interference and the light rays reinforce each other, and the workpiece and the table surface appear to be ordinarily illuminated. However, if the path of the light reflected from the workpiece surface differs in length from that of the light reflected from the mirror B by odd multiples of half wavelength, the workpiece surface appears to be dark because of destructive interference. The surface of the table also appears to be dark if the same situation is applied to it.

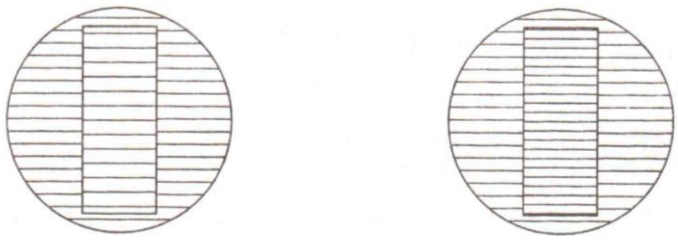
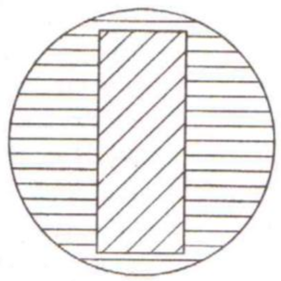


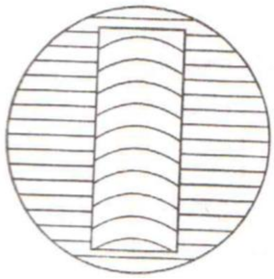
Figure 9.4 : Fringes Appearing on the Table and the Surface

In order that the operator can make measurement with an interferometer, the table is tilted slightly by a very small angle. This causes a series of interference fringes to appear on the surfaces of the workpiece and table as shown in Figure 9.3. If the workpiece is flat and parallel to the base plate, the fringe pattern will be straight, parallel and equally spaced. When the workpiece is flat but not parallel to the base plate, straight parallel fringes of different thickness will be obtained.

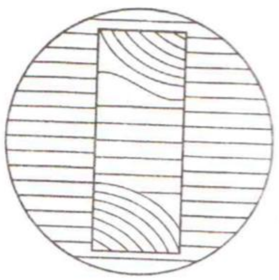
In case, the surface of the workpiece is at certain angle with the base plate, fringe pattern will be as shown in Figure 9.5(a). Here the error is indicated by the amount by which the fringes are out of parallelism with those on the base plate. If the workpiece is concave or convex, fringe pattern will be as shown in Figure 9.5(b). If fringe pattern is same as shown in Figure 9.5(c), it can be deduced that the surface is flat with slight rounding off at the corner.



(a)



(b)



(c)

Figure 9.5

9.6 LASER INTERFEROMETERS

The measuring capacity in interferometers of lamp of single wavelength as source of light is limited because of their low resolution and short measuring range. If the light source is replaced by a laser source, measurement can be done over a long distance because it facilitates to maintain the quality of interference fringes over long distance. Since laser is highly monochromatic coherent light source that follows all the principles of light, the fringes formed due to interference of laser are very sharp, accurate and precise.

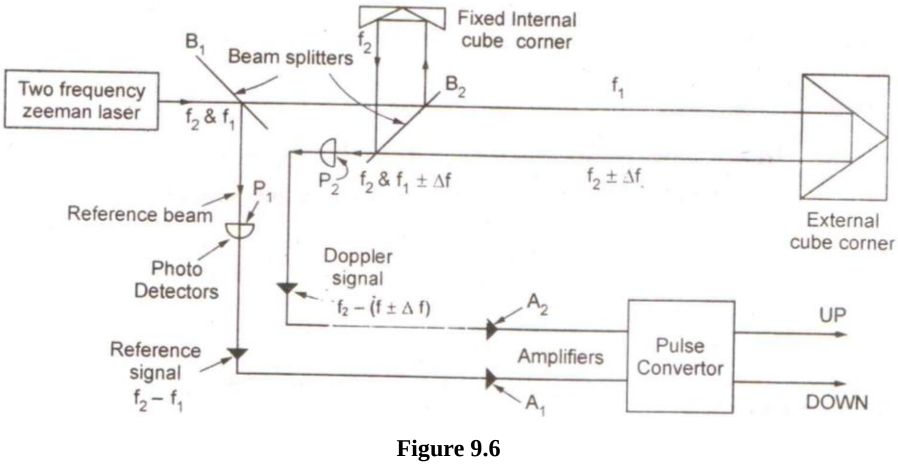


Figure 9.6

Figure 9.6 explains the operation of an Interferometer. It uses two-frequency laser system with opposite circular polarization. These beams get split up by beam splitter B_1, one part travel towards B_2 and the other towards external cube corner where the displacement has to be measured. Unlike Michelson Interferometer, mirror is not used as a reflector. Instead cube corner reflector is used. It reflects light parallel to its angle of incidence regardless of cube corner reflector's alignment accuracy. Beam splitter B_2 optically separates the frequency f_1, which alone is sent to the movable cube corner reflector. The second frequency f_2 from B_2 is sent to a fixed reflector. When these two light sources again meets once the cube corners reflect them to produce alternate light and dark interference patterns. When the movable reflector moves, the returning beam frequency will be Dopplar shifted slightly up or down by Δf. Thus, the light beams moving towards the photo-detector P_2 have frequencies f_2 and $f_1 \pm \Delta f$. P_2 changes these frequencies into electrical signal. Photo detector P_1 receives signal from the beam splitter B_1 and changes the reference beam frequencies f_1 and f_2 into electrical signal. An AC amplifier A_1 separates the frequency difference signal $f_2 - f_1$ and A_2 separates the frequency difference signal $f_2 - (f_1 \pm \Delta f)$. The pulse converter extracts Δf, one cycle per half wavelength of motion. The up and down pulses from the converter are counted electronically and displayed in analog or digital form on the indicator. From the value of Δf, the distance moved by the moving cube corner can be determined.

The uses of a laser interferometer are as given below.

(a) Since laser interferometer produce very thin, straight beam, they are used for measurements and alignment in the production of large machines.

(b) They are also used to calibrate precision machine and measuring devices.

(c) They can also be used to check machine set ups. A laser beam is projected against the work and measurements are made by the beam and displayed on a digital readout panel.

(d) Because of their very thin, straight beam characteristics, lasers are used extensively in constructions and surveying. They are used to indicate the exact location for positioning girders on tall building or establishing directional lines for a tunnel being constructed under a river.

(e) Laser interferometers can also be used in glass feature measurements.

The main advantages of a laser interferometer are as given below.

(a) Laser interferometers have high repeatability and resolution of displacement measurement.

(b) They give high accuracy (0.1 μ) of measurement.

(c) It facilitates to maintain long range optical path (60 m).

(d) Laser interferometers are easy to install.

(e) There is no chance deterioration in performance due to ageing or wear and tear.

SAQ 4

(a) Describe the principle and operation of a Michelson Interferometer.

(b) What are the uses and the advantages of a laser interferometer?

9.7 SUMMARY

In this unit, we have discussed the phenomenon of interference and its applications in measurements of various parameters. The unit begins with the discussion of conditions of interference and the light sources for this phenomenon to occur. Next, we have discussed how interferometry can be used for testing of flatness and surface contour. A typical interferometer is described after that. The unit finished with the discussion of the principle and working of a laser interferometer.

9.8 KEY WORDS

Interference : It is the phenomenon of light due to which light rays combines each other to give different amplitude and brightness.

Monochromatic Light : A light source, which has only one wavelength is called a monochromatic light source.

Phase : Phase of a light is defined as the angular status of the electromagnetic wave of which it consists of.

Optical Flat : An optical flat is a one-piece water white glass, generally used in measurement of flatness as a protector of camera lenses.

Flatness : Flatness of a surface is defined as the minimum separation of a pair of parallel planes, which will contain all the point on the surface.

Cube Corner : It is a reflecting device which reflects light parallel to its angle of incidence regardless of its alignment accuracy.

9.9 ANSWERS TO SAQs

SAQ 1

(a) See preceding text for answer.

(b) See preceding text for answer.

(c) See preceding text for answer

SAQ 2

(a) See preceding text for answer.

(b) See preceding text for answer

SAQ 3

(a) Let x be the distance between the adjacent fringes

$\therefore$ The Number of fringes $= \frac{L}{x} = 18$

$\therefore$ The difference between the height $d = L \frac{\lambda}{2x} = 18 \times \frac{0.664}{2} = 5.79\ \mu\text{m}$.

SAQ 4

(a) See preceding text for answer.

(b) See preceding text for answer.