ASSIGNMENT BOOKLET

BSCG/BAG/BSCFMT

REAL ANALYSIS

(1st January, 2026 to 31st December, 2026)

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THE PEOPLE'S
UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2026)

Dear Student,

Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1. On top of the first page of your answer sheet, please write the details exactly in the following format:

   ROLL NO. : ....

   NAME : ....

   ADDRESS : ....

   ....

   ....

   COURSE CODE : ....

   COURSE TITLE : ....

   STUDY CENTRE : ....   DATE ....

   PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2. Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3. Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4. Your answers should be precise.

5. While solving problems, clearly indicate which part of which question is being solved.

6. This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

7. This assignment is valid only up to 31st December, 2026. If you fail in this assignment or fail to submit it by the due date, then you need to get the assignment for the year 2027 and submit it as per the instructions given in the Programme Guide.

8. **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

9. **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

1. State whether the following statements are **true** or **false**. Justify your answers with a short proof or a counterexample (20)

    i) The negation of $p \vee (\sim q)$ is $q \to p$.

    ii) $\mathbb{Q} \backslash \mathbb{N}$ is countable.

    iii) The equation $x^3 - 2x + 3 = 0$ has a real root between -2 and 1.

    iv) Every increasing sequence has a convergent subsequence.

    v) The function defined as
    $$f(x) = \begin{cases} \frac{\sin 2x}{x}, & \text{if } x \neq 0 \\ 3, & \text{if } x = 0 \end{cases}$$

    has a removable discontinuity.

    vi) An integrable function can have finitely many points of discontinuity.

    vii) The series $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^n}$ is divergent.

    viii) The function f defined by $f(x) = \sin x (1 + \cos x)$, $x \in [0, 2\pi]$, has local maximum at $x = \frac{\pi}{3}$.

    ix) Every constant function on $[a, b]$ is Riemann Integrable.

    x) The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x - 2| + |x - 1|$ is differentiable at $x = 3$.

2. (a) Using mathematical induction, prove that the sum of the squares of first 'n' natural numbers is $\frac{n(n+1)(2n+1)}{6}$. (5)

   (b) State Cauchy's second theorem on limits. Use it to show that:

   $$\lim_{n \to \infty} \left[\frac{n}{[(n+1)(n+2)...(2n)]^{\frac{1}{n}}} \right] = \frac{e}{4}$$

   (5)

   (c) Check whether or not the sequence $(a_n)_{n \in \mathbb{N}}$, defined by:

   $$a_{n+1} = 2 - \frac{1}{a_n + 2} \ \forall n \geq 1$$

   and $a_1 = 2$ is convergent. (5)

3. (a) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^3 + 1}$. (4)

   (b) Find the radius of convergence of the series $\sum a_n x^n$, where $a_n = \frac{n!}{n^n}$. (3)

   (c) Show that the set :

   $$S = \left\{ \frac{1 + (-1)^n}{2} + \frac{1}{n} \ \bigg| \ n \in \mathbb{N} \right\}$$

   is not closed. (3)

4. (a) Show that the function $f :]0, \infty[\to \mathbb{R}$ defined as $f(x) = \frac{1}{x}$ is continuous on its domain, but not uniformly continuous there. Will the function be uniformly continuous on $[c, \infty[$ for any fixed $c > 0$? (6)

   (b) Examine the type of discontinuity of the function at $x = 0$ where

   $$f(x) = \begin{cases} x - |x|, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0. \end{cases}$$

   (4)

   (c) Check whether the following sets are open or closed or neither:

   (i) $\{3n : n \in \mathbb{N}\}$

   (ii) $]1, 6[\cup [2, 8]$ (5)

5. (a) Suppose that f and g are in $R[a, b]$. Then show that the function $f + g$ is in $R[a, b]$, and

   $$\int_a^b f + g = \int_a^b f + \int_a^b g.$$

   (5)

   (b) Discuss the convergence of the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$ by using the integral test. (5)

6. (a) Prove that $\sin x < x, x \in [0, \frac{\pi}{2}]$. (5)

   (b) Prove that every convergent sequence is Cauchy. (3)

   (c) Disprove the statement:

   $$\text{``} (x + y)^n = x^n + y^n \quad \forall n \in \mathbb{N}, \quad x, y \in \mathbb{Z} \text{''}$$

   by providing a suitable counter-example. (2)

7. (a) State Bolzano-Weirestrass theorem. Use it to check whether the set: (5)

   $$S = \left\{ \frac{1}{m} - \frac{1}{n} : m, n \in \mathbb{N} \right\}$$

   has a limit point.

   (b) Give one example for each of the following. Justify. (2)

   (i) A set which is neither open or closed.

   (ii) A set which has no limit point.

   (c) Check whether the set of integers is countable or not. (3)

8. (a) Let f be a differentiable function whose derivative never vanishes on $[a, b]$. Show that f is either strictly increasing or strictly decreasing. (5)

   (b) If an infinite series $\sum a_n$ converges, then show that $\lim_{n \to \infty} a_n = 0$. (5)

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