MMT-009

ASSIGNMENT BOOKLET

(Valid from 1st January, 2026 to 31st December, 2026)

M.Sc. (Mathematics with Applications in Computer Science)
Mathematical Modelling (MMT-009)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2026)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan., 2026 to 31st Dec., 2026**. If you have failed in this assignment or fail to submit it by Dec., 2026, then you need to get the assignment for the year 2027 and submit it as per the instructions given in the programme guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignments

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment (MMT – 009)

Course Code: MMT-009
Assignment Code: MMT-009/TMA/2026
Maximum Marks: 100

1. a) A company manufacturing soft drinks is thinking of expanding its plant capacity so as to meet future demand. The monthly sale for the past 6 years are available. **State, giving reasons,** the type of modelling you will use to obtain good estimates for future demand so as to help the company make the right decisions. Also state four essentials and four non-essentials for the problem. (6)

   b) Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz?

Portfolio	Expected return	Standard deviation
W	10%	25%
X	5%	7%
Y	17%	37%
Z	12%	13%

(3)

2. a) Let $G(t)$ be the amount of the glucose in the bloodstream of a patient at time t. Assume that the glucose is infused into the bloodstream at a constant rate of $g/\min$. At the same time, the glucose is converted and removed from the bloodstream at a rate proportional to the amount of the glucose present. If at $t = 0,\ G = 70g$ then
   i) formulate the model.
   ii) find $g(t)$ at any time t.
   iii) discuss the long term behavior of the model. (3)

   b) A tumour is developing from the organ of a human body with concentration 5.2×10^9 with growth and decay control parameters 7.2 and 2.7 respectively. In how many days the size of the tumor will be twice? (4)

3. a) Return distributions of the two securities are given below:

Return		Probabilities
X	Y	$p_{xj} = p_{yj} = p_j$
0.20	0.15	0.30
0.15	0.08	0.25
0.10	0.05	0.15
0.11	0.09	0.25

Find which security is more risky in the Markowitz sense. Also find the correlation coefficient of securities X and Y. (8)

   b) Let $P = (w_1,\ w_2)$ be a portfolio of two securities X and Y. Find the values of w_1 and w_2 in the following situations:
   i) $\rho_{xy} = -1$ and P is risk free.
   ii) $\sigma_x = \sigma_y$ and variance P is minimum.

iii) Variance P is minimum and $\rho_{xy} = -0.6$, $\sigma_x = 3$ and $\sigma_y = 5$. (6)

4. a) Companies considering the purchase of a computer must first assess their future needs in order to determine the proper equipment. A computer scientist collected data from seven similar company sites so that computer hardware requirements for inventory management could be developed. The data collected is as follows:

Customer Orders (in thousands)	Add-delete items (in thousands)	CPU time (in hours)
123.5	2.108	141.5
146.1	9.213	168.9
133.9	1.905	154.8
128.5	0.815	146.5
151.5	1.061	172.8
136.2	8.603	160.1
92.0	1.125	108.5

i) Find a linear regression equation that best fit the data. (4)
ii) Estimate the error variance for the regression model obtained in i) above. (2)

b) The population consisting of all married couples is collected. The data showing the age of 12 married couples is as follows:

Husband's age (years)	Wife's age (years)	Husband's age (years)	Wife's age (years)
32	27	51	50
25	30	48	46
36	34	37	36
72	65	50	42
37	37	51	46
36	38	36	35

i) Draw a scatter plot of the data (1)
ii) Write two important characteristics of the data that emerge from the scatter plot. (2)
iii) Fit a linear regression model to the data and interpret the result in terms of the comparative change in the age of husband and wife. (3)
iv) Calculate the standard error of regression and the coefficient of determination for the data. (3)

5. Consider a discrete model given by

$$N_{t+1} = \frac{r N_t}{1 + b N_{t-1}^2} = f(N_t), \ r > 1.$$

Investigate the linear stability about the positive steady state N^* by setting $N_t = N^* + n_t$. Show that n_t satisfies the equation

$$n_{t+1} - n_t + 2(r-1)r^{-1} n_{t-1} = 0.$$

Hence show that $r = 2$ is a bifurcation value and that as $r \to 2$ the steady state bifurcates to a periodic solution of period 6. (10)

6. a) The population dynamics of a species is governed by the discrete model

$$x_{n+1} = x_n \exp\left[r\left(1 - \frac{x_n}{K}\right)\right],$$

where r and k are positive constants. Determine the steady states and discuss the stability of the model. Find the value of r at which first bifurcation occurs. Describe qualitatively the behaviours of the population for $r = 2 + \varepsilon$, where $0 < \varepsilon << 1$. Since a species becomes extinct if $x_n \le 1$ for any $n > 1$, show using iterations, that irrespective of the size of $r > 1$ the species could become extinct if the carrying capacity $k < r\exp[1 + e^{r-1} - 2r]$. (7)

b) Do the stability analysis of the following model formulated to study the effect of toxicant on prey-predator population and interpret the solution.

$$\frac{dN_1}{dt} = r_0 N_1 - r_1 C_0 N_1 - bN_1 N_2$$

$$\frac{dN_2}{dt} = -d_0 N_2 - d_1 V_0 N_2 + \beta_0 b N_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - m_1 C_0$$

$$\frac{dV_0}{dt} = k_2 P - g_2 V_0 - m_2 V_0$$

$$\frac{dP}{dt} = Q - hP - kP(N_1 + N_2) + gC_0 N_1 + \ell V_0 N_2.$$

Where all the variables and constants are same as defined in the system (32)-(35) except for the following

$Q =$ constant input rate

$h =$ decay rate

$P(t) =$ environmental toxicant concentration

$k =$ ingestion rate of toxicant by the populations

$g, \ell =$ return rate of toxicant in the environment after the death of the populations, assuming that toxicant is non-degradable

$Q > 0, h, k, g, \ell$ are positive constants. (8)

7. Do the stability analysis of the following competing species system of equations with diffusion and advection

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} - v_1 \frac{\partial N_1}{\partial x}$$

$$\frac{\partial N_2}{\partial t} = -d_1 N_2 + C_1 N_1 N_2 + D_2 \frac{\partial^2 N_2}{\partial x^2} - V_2 \frac{\partial N_2}{\partial x}, \quad 0 \le x \le L$$

where V_1 and V_2 are advection velocities in x direction of the two populations with densities N_1 and N_2 respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, C_1 is the conversion rate. D_1 and D_2 are diffusion coefficients. The initial and boundary conditions are:

$N_i(x, 0) = f_i(x) > 0, \; 0 \le x \le L, \; i = 1, 2.$

$N_i = \overline{N}_i$ at $x = 0$ and $x = L \forall t, \; i = 1, 2.$

where $\overline{N}_i$ are the equilibrium solutions of the given system of equations.

Interpret the solution obtained and also write the limitations of the model. (10)

8. a) Maximize $Z = 5x_1 + x_2 + 3x_3$, subject to the constraints $-x_1 + 2x_2 + x_3 \leq 4$, $4x_2 - 3x_3 \leq 2$, $x_1 - 3x_2 + 2x_3 \leq 3$ and $(x_1, x_2, x_3) \geq 0$ and are integers. (8)

b) Ships arrive at a port at the rate of one in every 6 hours with exponential distribution of inter-arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 12 hours. If the average delay of ships waiting for berths is to be kept below 15 hours, how many berths should be provided at the port? (5)

c) A library wants to improve its service facilities in terms of the waiting time of its borrowers. The library has two counters at present and borrowers arrive according to Poisson distribution with arrival rate 2 every 10 minutes and service time follows exponential distribution with a mean of 15 minutes. The library has relaxed its membership rules and a substantial increase in the number of borrowers is expected. Find the number of additional counters to be provided if the arrival rate is expected to be twice the present value and the average waiting time of the borrower must be limited to half the present value. (7)