

MMTE-001

**ASSIGNMENT BOOKLET**

**M.Sc. Mathematics with Applications in Computer Science  
(MSCMACS)**

**GRAPH THEORY**

(1st January, 2026 to 31st December, 2026)



**School of Sciences  
Indira Gandhi National Open University  
Maidan Garhi, New Delhi-110068  
( 2026)**

Dear Student,

Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 20%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

### Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1. On top of the first page of your answer sheet, please write the details exactly in the following format:

---

ROLL NO. : .....

NAME : .....

ADDRESS : .....

.....

.....

.....

COURSE CODE : .....

COURSE TITLE : .....

STUDY CENTRE : ..... DATE .....

---

**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

2. Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3. Leave a 4 cm margin on the left, top and bottom of your answer sheet.
4. Your answers should be precise.
5. While solving problems, clearly indicate which part of which question is being solved.
6. This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**
7. This assignment is valid only up to 31st December, 2026. If you fail in this assignment or fail to submit it by the due date, then you need to get the assignment for the year 2027 and submit it as per the instructions given in the Programme Guide.
8. **You cannot fill the Exam form for this course till you have submitted this assignment. So solve it and submit it to your study centre at the earliest.**
9. **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

## Assignment

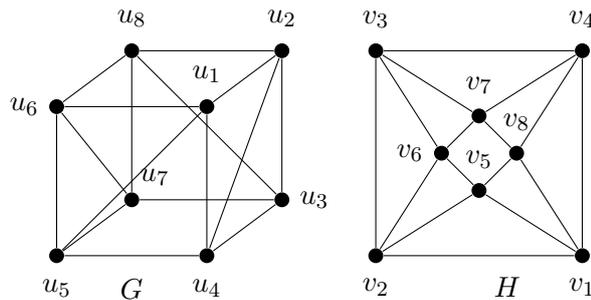
(To be done after reading the course material)

Course Code: MMTE-001

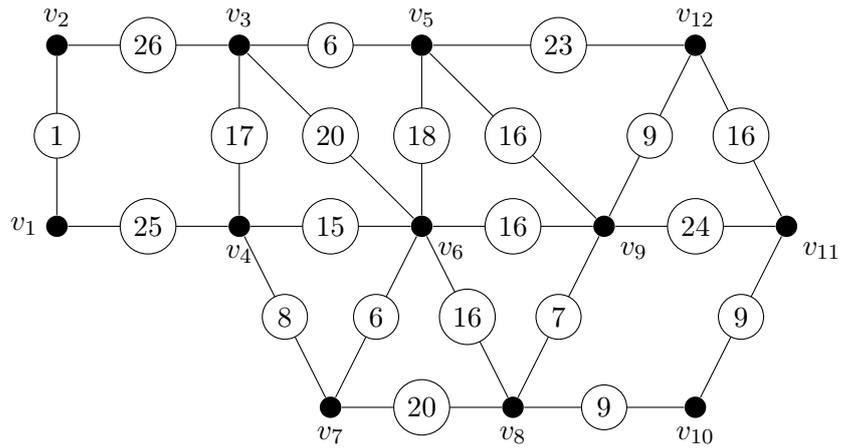
Assignment Code: MMTE-001/TMA/2026

Maximum Marks: 100

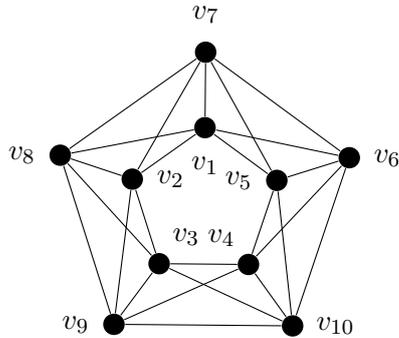
1. State whether the following statements are **true** or **false**. Justify your answers with a short proof or a counterexample (20)
  - i) There exists a simple graph on 10 vertices with degree sequence  $(4,4,3,3,3,2,2,1,1,1)$ .
  - ii) Every minimal vertex cut of a 2-connected graph induces a connected subgraph.
  - iii) If  $G$  is a connected graph with *diameter* 2, then  $\chi(G) \leq \Delta(G) + 1$ .
  - iv) Every bipartite graph is planar.
  - v) Every 4-regular bipartite graph admits a proper 2-edge-colouring.
  - vi) Every graph is isomorphic to its complement.
  - vii) The Petersen graph is non-Hamiltonian.
  - viii) There exists a triangle-free graph  $G$  with  $\chi(G) \geq 4$ .
  - ix) For every integer  $k \geq 4$ , any  $k$ -critical graph  $G$  satisfies  $|E(G)| \geq \frac{(k-1)|V(G)|}{2}$ .
  - x) The hypercube  $Q_n$  contains a *Hamiltonian cycle* for all  $n \geq 2$ .
  
2. (a) Prove or disprove that the complement of a 2-connected graph must be connected. (4)
  - (b) Let  $G$  be a simple graph with  $n \geq 2$  vertices. Prove that  $G$  contains at least two vertices of the same degree. (3)
  - (c) Check whether the following graphs  $G$  and  $H$  are isomorphic or not. Give a complete justification. (3)



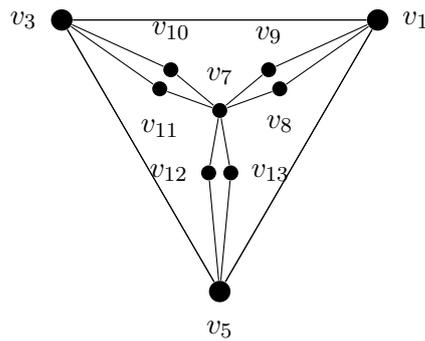
3. (a) Let  $G$  be a connected graph with  $n$  vertices and  $n + 1$  edges. Prove that  $G$  contains exactly two cycles. (3)
  - (b) Using a suitable algorithm, find a minimum weight spanning tree of the given weighted graph. Show all intermediate steps. (4)



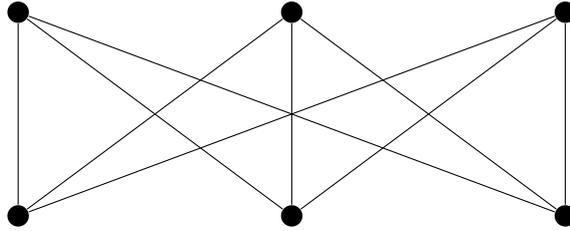
- (c) Prove that every maximum matching in a graph  $G$  is a maximal matching, but the converse need not be true. (4)
- (d) Determine the chromatic number and the edge-chromatic number of the following graph. (4)



4. (a) Determine the number of spanning trees of the given graph, clearly stating the theorem or method used. (4)



- (b) Solve the Chinese Postman Problem for the weighted graph given in 3(b). (5)
- (c) Prove that every 3-chromatic graph contains a cycle of odd length. (3)
- (d) Prove that every planar graph has a vertex of degree at most 5. (3)
5. (a) Verify Euler's formula for the given planar graph. (3)

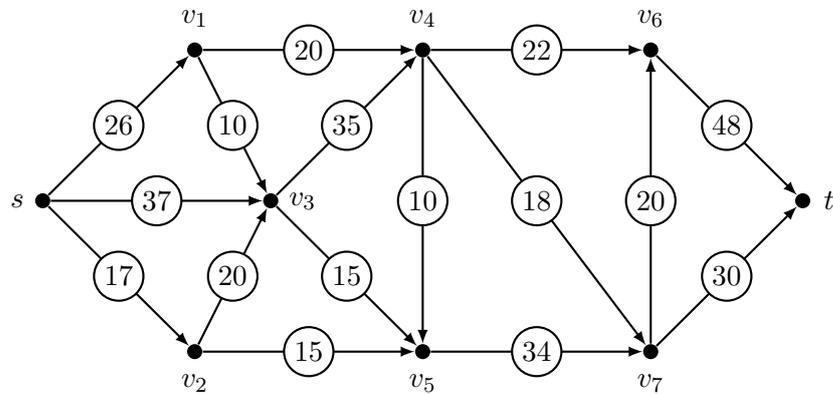


(b) Show that the number of leaves in an  $n$ -vertex complete  $k$ -ary tree is  $(nk - n + 1)/k$ . (4)

(c) Let  $G$  be a planar graph with  $n \geq 3$  vertices. Prove that  $|E(G)| \leq 3n - 6$ . (4)

(d) Let  $G$  be a connected graph. Define the eccentricity  $e(v)$  of a vertex  $v$ . Define the radius and diameter of  $G$ , and show that  $rad(G) \leq diam(G) \leq 2 \times rad(G)$ . (4)

6. (a) Determine the maximum possible flow that can pass through the given network. Also, describe a flow achieving this maximum value. (4)

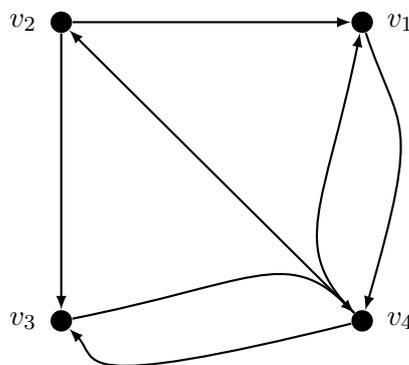


(b) State and prove Hall's Marriage Theorem. Illustrate the theorem with a suitable example. (5)

(c) Let  $G$  be a connected graph. Prove that  $G$  is bipartite if and only if its vertex set can be partitioned into two independent sets. (4)

(d) Give an example of a graph  $G$  with 5 blocks and 4 cut-vertices. (2)

7. (a) Let  $G$  be a connected graph. Define the line graph  $L(G)$ . Find the line graph of the given graph and determine the number of vertices and edges in  $L(G)$ . (4)



(b) Define thickness and crossing number. Find the thickness and crossing number of the graph  $G$  given in Q 2(c). (3)

- (c) Draw the Cartesian product graph  $C_3 \times P_4$ . Clearly, label all vertices and indicate the adjacency relations in the product. (3)