MMTE-005

ASSIGNMENT BOOKLET

M.Sc. (Mathematics with Applications in Computer Science)

CODING THEORY

(Valid from January 1, 2026– December 31, 2026)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi 110068
2026

Dear Student,
Please read the section on assignments in the Programme Guide that we sent you after your enrolment. As you may know already from the programme guide, the continuous evaluation component has 20% weightage. This assignment is for the continuous evaluation component of the course.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. : . . . .
NAME : . . . .
ADDRESS : . . . .
. . . .
. . . .
. . . .

COURSE CODE : . . . .
COURSE TITLE : . . . .
STUDY CENTRE : . . . . **DATE** . . . .

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) Write all the answers in your own words. Do not copy from the internet, from your fellow students or from any other source. If your assignment is found to be copied, it will be rejected.

7) This assignment is valid only up to 31st December, 2026. If you fail in this assignment or fail to submit it by 31st December, 2026, then you need to get the assignment for 2027 and submit it as per the instructions given in the Programme Guide.

8) It is **compulsory** to submit the assignment before you submit your examination form.

9) For any doubts, clarifications and corrections, write to `svenkat@ignou.ac.in`.

We **strongly** suggest that you retain a copy of your answer sheets.

Wish you good luck.

Assignment

Course Code: MMTE-005
Assignment Code: MMTE-005/TMA/2026
Maximum Marks: 100

1) Which of the following statements are true and which are false? Justify your answer with a short proof or a counterexample. (10)

i) If the weight of each element in the generating matrix of a linear code is at least r, the mininum distance of the code is at least r.

ii) There is no linear self orthogonal code of odd length.

iii) There is no 3-cyclotomic coset modulo 121 of size 25.

iv) There is no duadic code of length 15 over $\mathbf{F}_2$.

v) There is no LDPC code with parameters $n = 16$, $c = 3$ and $r = 5$.

2) a) Which of the following binary codes are linear?

i) $\mathscr{C} = \{(0,0,0,0),(1,0,1,0),(0,1,1,0),(1,1,1,0)\}$

ii) $\mathscr{C} = \{(0,0,0),(1,1,0),(1,0,1),(0,1,1)\}$

Justify your answer. (3)

b) Find the minimum distance for each of the codes. (4)

c) For each of the linear codes, find the degree, a generator matrix and a parity check matrix. (3)

d) For a linear code $\mathscr{C}$ with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

find the parity check matrix. Check that any two columns of the parity check matrix are linearly independent and there are three columns that are linearly dependent. What is the minimum distance of $\mathscr{C}$? (5)

3) a) Find the parity check matrix of the code $\overline{\mathscr{H}}_3$. Decode the following vectors

i) (1 1 1 0 0 0 0 1)

ii) (1 0 1 1 0 0 1 0)

iii) (0 0 1 1 0 1 0 0)

iv) (0 1 1 1 1 0 0 1)

(10)

b) Find the parity check matrix of the code $\mathscr{H}_{2,5}$.

c) Let $\mathscr{C}_1$ and $\mathscr{C}_2$ be two binary codes with generator matrices

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},\ G_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix},$$

respectively.

i) Find the minimum distance of both the codes.

Table 1: Table for $\mathbf{F}_16$.

0000	0	1000	α^3	1011	α^7	1110	α^{11}
0001	1	0011	α^4	0101	α^8	1111	α^{12}
0010	α	0110	α^5	1010	α^9	1101	α^{13}
0100	α^2	1100	α^6	0111	α^{10}	1001	α^{14}

ii) Find the generator matrix of the code

$$\mathscr{C} = \{(\mathbf{u}|\mathbf{u}+\mathbf{v})|\mathbf{u} \in \mathscr{C}_1, \mathbf{v} \in \mathscr{C}_2\}$$

obtained from $\mathscr{C}_1$ and $\mathscr{C}_2$ by $(\mathbf{u}|\mathbf{u}+\mathbf{v})$ construction. Also, find the minimum distance of $\mathscr{C}$. (5)

4) a) For the binary, $(6,3)$ linear code $\mathscr{C}$ with generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$, prepare a standard array for decoding. Use it to decode the vectors $(1,1,1,0,1,1)$, and $(1,1,0,1,1,1)$. (7)

b) Prepare a syndrome table for $\mathscr{C}$ in part a) and decode the vectors $(1,1,1,1,0,1)$ and $(0,1,0,1,1,1)$. (5)

c) The aim of this exercise is to show that every binary repetition code of odd length is perfect.

i) Find the value of t and d for a binary repetition code of length $2m+1$, $m \in \mathbf{N}$. (2)

ii) Show that (3)

$$\sum_{i=0}^{m} \binom{2m+1}{i} = 2^{2m}$$

(**Hint:** Start with the relation

$$2^{2m+1} = \sum_{i=0}^{2m+1} \binom{2m+1}{i})$$

iii) Deduce that every repetiition code of odd length is perfect. (2)

5) a) Let $\mathscr{C}$ be the ternary $[8,3]$ narrow-sense BCH code of designed distance $\delta = 5$, which has defining set $T = \{1,2,3,4,6\}$. Use the primitive root 8th root of unity you chose in 4a) to avoid recomputing the the table of powers. If

$$g(x) = x^5 - x^4 + x^3 + x^2 - 1$$

is the generator polynomial of $\mathscr{C}$ and

$$y(x) = x^7 - x^6 - x^4 - x^3$$

is the received word, find the transmitted codeword. Use the following table in 1. (7)

b) Let $\mathscr{C}$ be the [5, 2] binary code generated by

$$G = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Find the weight enumerator $W_{\mathscr{C}}(x,y)$ of $\mathscr{C}$. Use McWilliams identity to find the weight enumerator of $\mathscr{C}^\perp$. Verify your answer by finding the generator matrix of $\mathscr{C}$ and finding the weight distribution of $\mathscr{C}^\perp$. (8)

c) Let $\mathscr{C}$ be a cyclic code of length eight over $\mathbf{F}_3$with generator polynomial $\overline{2}+x+x^3+x^4$. Find the generator matrix of $\mathscr{C}$, the generator polynomial of $\mathscr{C}^{\perp}$ and the parity check matrix of $\mathscr{C}^{\perp}$. (6)

6) a) Factor x^5-1 over $\mathbf{F}_2$. Give the generator polynomials of all cyclic codes of length five over $\mathbf{F}_2$. (10)

b) Factor x^8-1 over $\mathbf{F}_5$. Give the generator polynomials of all cyclic codes of length eight over $\mathbf{F}_5$ (10)