No. of Printed Pages : 5 **BEY–018**

B. SC. (APPLIED SCIENCE—ENERGY) (BSCAEY)

Term-End Examination

December, 2024

BEY–018 : LINEAR ALGEBRA AND CALCULUS

Time : 3 Hours *Maximum Marks : 70*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***six*** *questions from the remaining question nos. 2 to 9.*

(iii) Use of scientific (non-programmable) calculator is allowed in exam.

(iv) Symbols have their usual meanings.

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1. (a) Using the properties of determinants, show that : 2

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix} = 0$$

(b) Show that the three points with position vectors $2\hat{i} + 3\hat{j}$, $3i + \frac{9}{4}\hat{j}$ and $5\hat{i} + \frac{3}{4}\hat{j}$ are collinear. 2

(c) Find the interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly decreasing. 2

(d) For the following differential equation, determine its order and degree. Also determine whether the equation is linear or not : 2

$$\frac{d^2y}{dx^2} + \cos(x + y) = \sin x .$$

(e) Find the radius of convergence R of the power series $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$. 2

2. Solve the following system of equations using Cramer's rule : 10

$$2x + 3y + z = 4$$

$$x - y + 2z = 9$$

$$3x + 2y - z = 1$$

3. (a) Find the points where the tangent to the circle $x^2 + y^2 = 16$ is parallel to the line $\sqrt{3}x - y + 6 = 0$. 5

(b) A balloon which remains spherical has a diameter $\frac{3}{2}(2x+3)$. Determine the rate of change of volume w.r.t. x. 5

4. (a) Show that the function :

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{, if } (x, y) \neq (0, 0) \\ 0 & \text{, if } (x, y) = (0, 0) \end{cases}$$

is discontinuous at the origin. 5

(b) For $u = x^3y^2 - y^5.x$, show that : 5

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$$

5. (a) Solve the following differential equation : 5

$$x^2 y \frac{dy}{dx} = (1 + x) \operatorname{cosec} y$$

(b) Use Picard's iterative method to find the first two approximations for the solution of the non-linear IVP : 5

$$\frac{dy}{dx} = 3xy + y^2, \qquad y(0) = 1.$$

6. (a) Show that the two non-zero differentiable functions $f(x)$ and $g(x)$, defined on an interval I, are linearly independent on I if and only if the Wronskian : 6

$$W(f, g) = 0 \qquad \text{for all } x \in I.$$

(b) Show that the functions e^{-x} and xe^{-x} form a basis for $y'' + 2y' + y = 0$ and then write its general solution. 4

7. (a) Discuss the maximum and minimum of $x^2 + y^2 + 6x + 12$. 6

(b) If $x = r \cos \theta$, $y = r \sin \theta$, $z = z$, find : 4

$$\frac{\partial(x, y, z)}{\partial(r, \theta, z)}.$$

8. (a) Verify Lagrange's mean value theorem for $f(x) = x^2 + 2x + 3$, $x \in [4, 6]$. 5

(b) Show that $x = 0$ is an irregular singular point of $(x \sin x)\, y'' + 4y' + 20xy = 0$. 5

9. (a) Write a short note on Bessel's equation. 5

(b) For the equation :

$$y'' + p_1(x)y' + p_2(x)y = 0$$

where $p_1(x)$ and $p_2(x)$ are continuous functions of x in an interval I, show that there exists a basis on I. 5

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