No. of Printed Pages : 4 **MMT–002**

M. SC. (MATHEMATICS WITH APPLICATIONS TO COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

December, 2024

MMT–002 : LINEAR ALGEBRA

Time : 1½ Hours *Maximum Marks : 25*

Weightage : 70%

Note : *(i) There are five questions in this paper.*

*(ii) The **fifth** question is compulsory.*

*(iii) Do any **three** questions from Q. No. **1** to Q. No. **4**.*

*(iv) Use of calculator is **not** allowed.*

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1. (a) Consider : 2

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x+2y \\ x+3y \end{bmatrix}$$

Find the matrix of T with respect to the standard basis of $\mathbf{R}^2$ and then use the change of basis formula to find its matrix with respect to the basis :

$$\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$$

(b) Find the QR decomposition of : 3

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

2. Solve the system of differential equations : 5

$$\frac{dy(t)}{dt} = Ay(t)$$

where :

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}, \; y(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

3. (a) Determine if the following matrix is positive definite or not : 2

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(b) The owl (O) and rat (R) populations in a forest are governed by the equations : 3

$$O_{k+1} = 0.5O_k + 0.4R_k$$

$$R_{k+1} = -0.104O_k + 1.1R_k$$

What is the ratio of the population in the long-run ?

4. Find the singular value decomposition of : 5

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}$$

5. Which of the following statements are true and which are false ? Give reasons for your answer : 10

(i) If the characteristic polynomial of a matrix in $M_3(\mathbf{R})$ is $(x-1)(x-2)(x-3)$, then its Jordan canonical form is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(ii) If a matrix A has spectral radius 0, then A = 0.

(iii) If all the entries of matrix A are positive, then A is a positive definite matrix.

(iv) The sum of any two unitary matrices is unitary.

(v) The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ has a unique generalised inverse.

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