 MMT–003

M. SC. (MATHEMATICS WITH APPLICATION IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2024

MMT–003 : ALGEBRA

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Answer any* ***four*** *questions from Question No.* ***2*** *to* ***6****. Calculators are not allowed. Show all the steps involved in every solution you do.*

1. Which of the following statements are true and which are false ? Justify your answers with a short proof or counter-example, whichever is appropriate. 10

   (i) If a group of order 32 acts on a set with 31 elements, then there must be at least one singleton orbit.

(ii) In the ring of Gaussian integers $\mathbf{Z} + \mathbf{Z}i$ (subring of $\mathbf{C}$), | | is an irreducible element.

(iii) If p is a prime congruent to 2 (mod 3), then there exists an integer x such that $x^2 = -3 \pmod{p}$.

(iv) ($\mathbf{N}$, .) is a finitely generated semi-group.

(v) If k is a field, then $k \times k$ is a UFD.

2. Let $K = \mathbf{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$. Show that K is a Galois extension of $\mathbf{Q}$ and the Galois group $\mathbf{G}(K/\mathbf{Q})$ is isomorphic to $\mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2$. 10

3. (a) Determine how many isomorphism classes of abelian groups of order 36 are there. Justify your answer. Also, write the groups from each of the isomorphism classes. 4

(b) Find the integer x, $1800 < x < 3600$, such that $x \equiv 2 \pmod 9$, $x \equiv 3 \pmod{25}$ and $x \equiv 4 \pmod 8$. 4

(c) Check whether $\mathbf{Q}(i)$ is algebraically closed or not. 2

4. Show that if G is a non-abelian group of order 8, then o (Z(G)) = 2. Hence show that G is either isomorphic to D_8 or Q_8. 10

5. (a) Find the number of distinct monic irreducible polynomials in $\mathbf{Z}_5[x]$ of degree 3. 5

   (b) Prove that SU_2 and $S^3 \subseteq \mathbf{R}^4$ are isomorphic. 5

6. (a) Check whether a group of order 12 is simple or not. 3

   (b) Let A be a finite abelian group and B be a finitely generated free abelian group. Find Tor(A), Tor(B) and show that Tor(A × B) is isomorphic to A. 4

   (c) Check whether or not a field is a Euclidean domain. 3

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