

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE) [M. SC. (MACS)]**

Term-End Examination

December, 2024

MMT-004 : REAL ANALYSIS

Time : 2 Hours

Maximum Marks : 50

Note : *Question No. 1 is compulsory. Attempt any
four questions out of question nos. 2 to 6.
Use of calculator is not allowed.*

1. State whether the following statements are
True or False. Give reasons for your answers :

$$5 \times 2 = 10$$

- (a) The product of two discrete metrics is a
discrete metric.

- (b) The set of integers is neither dense nor nowhere dense in $\mathbf{R}$ with respect to the usual metric on $\mathbf{R}$.
- (c) $\mathbf{R}^2/\{(0, 1)\}$ is compact but not connected.
- (d) For the function :

$$f(x, y, z) = x^2y + y^2z + z^2,$$

$$f''(1, 1, -1/2) \text{ is } \begin{bmatrix} 2 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$

- (e) All continuous real valued functions defined on $[0, 1]$ are measurable but not integrable.
2. (a) Find the interior, closure and boundary of the set :

2

$$B = \{(x, y) \in \mathbf{R}^2 : x = 0\}$$

- (b) Define uniform continuity of functions between metric spaces. Prove that a uniformly continuous image of a Cauchy sequence is a Cauchy sequence.

4

- (c) For the function $F : \mathbf{R}^4 \rightarrow \mathbf{R}^3$ given by :

$$F(x, y, z, w) = (x^2y, xyz, x^2 + y^2 + zw^2)$$

Write down the component functions. Find the derivative of F at $(0, 0, 0, 0)$. 4

3. (a) Prove that a compact subset of a metric space is closed and bounded. Is the converse true ? Justify. 4

- (b) Prove that the Lebesgue measure is countably additive. 3

- (c) Check whether the function : 3

$$f(x, y) = (x^2 - y^2, 2xy)$$

is locally invertible on $E = \{(x, y) \mid x > 0\}$.

4. (a) Prove that a non-empty subset of $\mathbf{R}$ is connected if and only if it is an interval ($\mathbf{R}$ with the usual metric). 4

- (b) Find and classify the stationary points of the function $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ given by : 3

$$f(x, y) = (y - x^2)(y - 2x^2)$$

- (c) Suppose f is a non-negative measurable function and $E \in \mathcal{M}$. Define $\int_E f dm$. For

$A, B \in \mathcal{M}$ with $A \cap B = \emptyset$, prove that : 3

$$\int_{A \cup B} f dm = \int_A f dm + \int_B f dm$$

5. (a) Prove that a continuous function from a compact metric space to any other metric space is uniformly continuous. 4

- (b) (i) State and prove the Monotonic convergence theorem. 4

(ii) Does the sequence :

$$f_n = \chi_{[n, n+1]}, n=1, 2, \dots$$

satisfy all the conditions of the above theorem ? Check. 2

6. (a) For a real function $f \in L, [-\pi, \pi]$, define the n th Fourier coefficient of f , the exponential and the trigonometric form of the Fourier series of f .

Find the Fourier series of the function : 4

$$f(t) = \begin{cases} 0, & -\pi \leq t < 0 \\ 1, & 0 \leq t \leq \pi \end{cases}$$

- (b) State the Riemann-Lebesgue lemma. 2
- (c) Find the extreme values of the function : 4

$$Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

Subject to the constraint :

$$x_1 + x_2 + x_3 = 20,$$

$$x_1, x_2, x_3 \geq 0.$$

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