

No. of Printed Pages : 4

MMT-005

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE) [M. SC. (MACS)]**

Term-End Examination

December, 2024

MMT-005 : COMPLEX ANALYSIS

Time : 1½ Hours

Maximum Marks : 25

***Note : Question No. 1 is compulsory. Attempt any
three questions from Question No. 2 to 5.***

Use of calculator is not allowed.

1. State, giving reasons, whether the following
statements are True or False : $5 \times 2 = 10$

- (a) The intersection of two open connected sets
of complex numbers is an open connected
set.

- (b) $\cos z$ is a bounded function.
- (c) All the singular points of the function :

$$f(z) = \frac{1}{(2z-1)(3z-2)}$$

are isolated singular points.

- (d) The radius of convergence of the power series :

$$\sum_{n=1}^{\infty} \frac{n^{50}(z-5)^n}{n!}$$

is 5.

- (e) The function $f(z) = 9z - \frac{4}{z}$ is conformal at

$$z = \pm \frac{2}{3}i.$$

2. (a) Find the harmonic conjugate of the function :

2

$$u(x, y) = e^{-x} \cos y$$

- (b) Find the Laurent's series expansion in powers of z for the function : 3

$$f(z) = \frac{1}{z^2 - 7z + 12}$$

which is valid in the region $|z| > 4$.

3. (a) Let function f be analytic everywhere inside and on a positive oriented circle C_R centred at Z_0 with radius R . Show that : 2

$$|f^{(n)}(z_0)| \leq \frac{n! M}{R^n}, \quad n = 1, 2, 3, \dots$$

where M is maximum value of $f(z)$ on the circle $|z - z_0| = R$.

- (b) Find, for the following function : 3

$$f(z) = \frac{\cos(z^2) - 1}{z^6}$$

- (i) The principal part of $f(z)$ at $z = 0$.
- (ii) The nature of the singular point $z = 0$.
- (iii) The residue at $z = 0$.

4. (a) Find the Mobius transformation which maps $-i$, 1 , i to $-i$, 0 and i respectively. 3
- (b) Find all the solutions of : 2

$$e^z = (1 + \sqrt{3}i)$$

5. Evaluate the following improper integral : 5

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$$

using the method of contour integration.

× × × × × × ×