No. of Printed Pages : 7 **MMT–009**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. Sc. (MACS)]

Term-End Examination

December, 2024

MMT–009 : MATHEMATICAL MODELLING

Time : 1½ Hours *Maximum Marks : 25*

Weightage : 70%

Note : *Attempt any **five** questions. Use of scientific (non-programmable) calculator is allowed.*

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1. (a) Explain each of the following with examples : 2

 (i) Reaction-diffusion model *Vs.* Advection-reaction-diffusion model.

 (ii) Variational matrix of a system of n-dimensional equations.

(b) Find a linear demand equation that best fits the following data and use it to predict annual sales of homes priced at ₹ 14,00,000 : 3

x = price (lakhs of ₹)	16	18	20	22	24	26	28
y = sales of new homes this year	126	103	82	75	82	40	20

2. Do the stability analysis of the following model formulated to study the effect of toxicant on prey-predator population : 5

$$\frac{dN_1}{dt} = r_0 N_1 - r_1 C_0 N_1 - b N_1 N_2$$

$$\frac{dN_2}{dt} = -d_0 N_2 - d_1 V_0 N_2 + \beta_0 b N_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - m_1 C_0$$

$$\frac{dV_0}{dt} = k_2 P - g_2 V_0 - m_2 V_0$$

Here r_0, d_0, b, β_0, m_1, m_2, P, k_1, k_2, g_1, g_2, r_1 and d_1 are all positive constants.

$N_1(t)$ = Density of prey population

$N_2(t)$ = Density of predator population

$C_0(t)$ = Concentration of the toxicant in the individuals of the prey population

$V_0(t)$ = Concentration of the toxicant in the individuals of the predator population

r_0 = Growth rate or birth rate

d_0 = Death rate

b = Predation rate

β_0 = Conversion coefficient

m_1, m_2 = Depuration rates

k_1, k_2 = Uptake rates

g_1, g_2 = Loss rates

r_1 = Death rate due to C_0

d_1 = Death rate due to V_0

P = Constant environmental toxicant concentration

3. (a) A telephone exchange has two long distance operators. The telephone company observes that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes. 3

(i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day ?

(ii) If the subscriber will wait and serviced in turn, what is the expected waiting time ?

(b) Find the number of quantities required for estimating the expected return and standard deviation for 300 securities in Markowitz model. How many estimates are required for these securities while using single index Sharpe model ? 2

4. Solve the integer programming problem : 5

Max. : $Z = 7x_1 + 9x_2$

subject to : $-x_1 + 3x_2 \le 6$

$7x_1 + x_2 \le 35$

$x_1 \ge 0, x_2 \ge 0$ and integers.

5. (a) A discrete model for a population N_t consists of :

$$N_{t+1} = \frac{rN_t}{1 + bN_t^2}$$

where t is the discrete time and r and b are positive parameters. Determine the non-negative steady state and discuss the linear stability of the model. 3

(b) What are the factors to describe the mathematical models in population dynamics ? Write any *four* of them with explanation. 2

6. Consider the budworm population dynamics governed by the equation $\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - x,$ where k, the carrying capacity and r, the birth rate of the budworm population $x(t)$, are positive parameters. Find the steady states and use the perturbation to do the stability analysis of the equation for $0 < r < 1$. 5

7. (a) The deviation $g(t)$ of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation :

$$4\frac{d^2g}{dt^2} + 8\alpha\frac{dg}{dt} + (2\alpha)^2 g = 0$$

for α, a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identify the type (overdamped) of this differential equation. Find the condition on α for which the patient is normal. 2½

(b) In a tumour region, the control parameters of growth and decay are in 2 : 1. Emigration occurs at a constant rate of 3×10^3 cells per month. There is decay of 35 cells every month. Use these assumptions to formulate the logistic model of tumour size. Solve the formulated equation if the initial size of the tumour is 5×10^6 cells. When does the proliferation of tumour cells stabilize for this model ? 2½

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