

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE) [M. SC. (MACS)]**

Term-End Examination

December, 2024

MMTE–001 : GRAPH THEORY

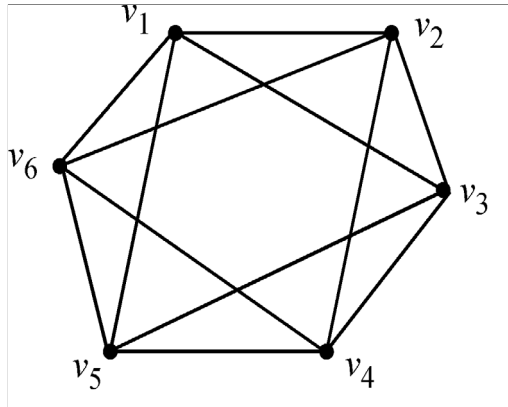
Time : 2 Hours

Maximum Marks : 50

Note : *Question No. 1 is compulsory. Answer any
four questions from question no. 2 to 7. Use
of calculator is **not** allowed.*

1. State whether the following statements are True or False. Justify your answers with a short proof or a counter-example : $5 \times 2 = 10$
- (i) Every complete graph $K_n, n \geq 3$ is Eulerian.
- (ii) For any two graphs G, H ; $G \times H \cong H \times G$.
- (iii) Every wheel graph W_n has a perfect matching.

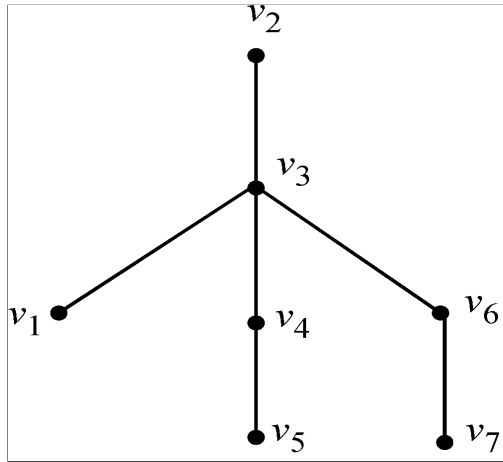
- (iv) Every tree is a bipartite graph.
- (v) For every graph G ; $w(G) \leq \chi(G)$.
2. (a) Write the adjacency matrix of the following graph : 3



- (b) Draw a 5-regular graph on 8 vertices. 3
- (c) Prove that every tree on n vertices has $(n - 1)$ edges. 4
3. (a) Prove that in any graph, the sum of all the vertex-degrees is equal to twice the number of edges. 3
- (b) Prove that : 3

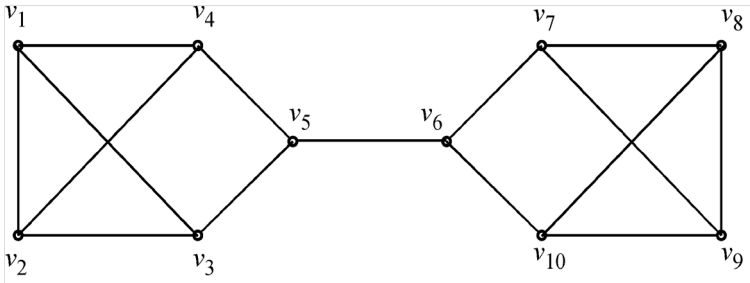
$$\chi(G) \geq \frac{n}{\alpha(G)}$$

- (c) Find the eccentricity of each vertex in the following tree. Also find its diameter and radius. : 4

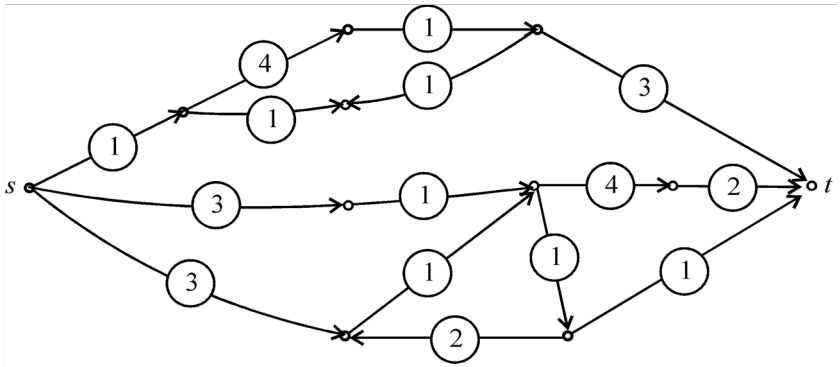


4. (a) Prove or disprove : The complement of a disconnected graph must be connected. 3
- (b) Prove that in any bipartite graph all cycles have even length. 4
- (c) Define Perfect Matching in a graph with an example. 3
5. (a) State and prove Ore's theorem. 6
- (b) Show that for any graph G , $\beta(G) \geq \alpha'(G)$.
Give an example of a graph for which the equality holds. 4

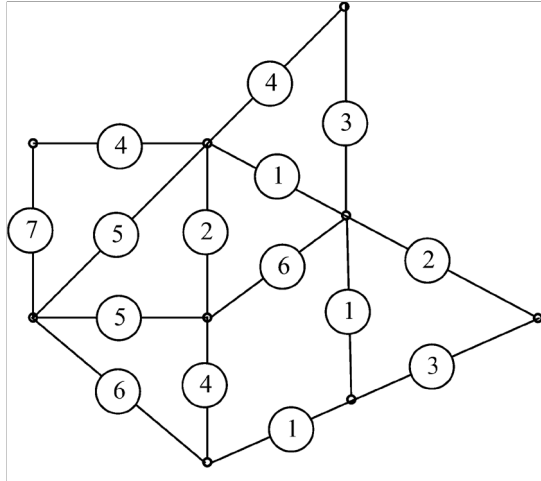
- 5



- 5



7. (a) Find a minimum weight spanning tree in the following weighted graph : 5



- (b) Let G be a 3-regular graph with a cut-vertex. Show that G has at least 3 blocks. 5

× × × × × × ×