No. of Printed Pages : 4 **MMTE–005**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2024

MMTE–005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) There are six questions in this paper.*

(ii) The ***sixth*** *question is compulsory.*

(iii) Do any ***four*** *questions from question nos.* ***1*** *to* ***5****.*

(iv) Use of calculator is not allowed.

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1. (a) Define the following, giving an example of each : 6

(i) Self-dual code

(ii) Hamming weight of a codeword

(iii) Generator matrix

(b) Compute the 2-cyclotomic cosets modulo 19. 4

2. (a) (i) Construct the parity check matrix of the binary Hamming code H_8 of length 15. 3

(ii) Using the above parity check matrix decode the vector 001000001100100. 3

(b) Find the g.c.d. of $x^4 + x^3 + x^2 + x + 2$ and $x^4 + 2x^2 + x + 1 \in \mathbf{F}_3[x]$. 4

3. (a) Let C be the narrow-sense binary BCH code of designed distance $\delta = 5$ which has a designing set $T = \{1, 2, 3, 4, 6, 8, 9, 12\}$. Let α be a primitive 15th root of unity, where $\alpha^4 = \alpha + 1$ and let the generator polynomial of C be $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. If $y(x) = x + x^4 + x^7 + x^8 + x^{11} + x^{12} + x^{13}$ is received, find the transmitted code-word.

You can use the following table : 6

0000	0	1000	α^3	1011	α^7	1110	α^{11}
0001	1	0011	α^4	0101	α^8	1111	α^{12}
0010	α	0110	α^5	1010	α^9	1101	α^{13}
0100	α^2	1100	α^6	0111	α^{10}	1001	α^{14}

(b) The systematic generator matrix for a [5, 2] binary, linear code is : 4

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Find the standard array for decoding.

4. (a) Find a generator polynomial and parity check polynomial of a [7, 4] binary cyclic code. Find the generator matrix and parity check for the code. 4

(b) Construct the generator matrix $G(1, 3)$ of the Reed-Muller code $R(1, 3)$. 3

(c) Prove that, if the minimum distance of a code C is d, the minimum distance of the extended code $\widehat{C}$ is d or $d+1$. 3

5. (a) Let C be a cyclic code of length n over $\mathbf{F}_q$ with defining set T. Suppose C has minimum weight d. Assume T contains $\delta - 1$ consecutive elements for integer δ. Then $\delta \geq d$. 5

(b) Let C be the [5, 2]-binary code generated by :

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Find the weight distribution of C. Find the weight distribution of the dual code $C^{\perp}$ using the MacWilliams identity. 5

6. Which of the following statements are true and which are false ? Justify your answer with a short proof or a counter-example, whichever is appropriates : 10

(a) Every self-orthogonal code is self-dual.

(b) The code C = {00000, 11111} can correct 3 errors.

(c) There is a 2-cyclotomic set of modulo 31 of size 7.

(d) The Reed-Muller code $R(1, 3)$ is a self-dual code.

(e) The code C = {0000.0100, 1000, 0010} is a cyclic code.

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