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MPH–001

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

December, 2024

MPH–001 : MATHEMATICAL METHODS IN PHYSICS

Time : 2 Hours *Maximum Marks : 50*

Note : *Attempt any **five** questions. Marks are indicated against each question. Symbols have their usual meanings. You may use calculator.*

1. Using the generating function for Hermite polynomials : 7+3

$$g(x, t) = e^{2xt-t^2} = \sum_{n=0}^{\infty} \mathrm{H}_n(x)\frac{t^n}{n!}$$

obtain (i) the recurrence relation :

$$\mathrm{H}_{n+2}(x) = 2x\,\mathrm{H}_{n+1}(x) - 2(n+1)\mathrm{H}_n(x)$$

(ii) establish the relation between $H_n(x)$ and $H_n(-x)$ as :

$$H_n(-x) = (-1)^n H_n(x)$$

2. (a) The one-dimensional differential equation for the function E_y is given by : 5

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$

Solve this equation if $E_y = 0$ at $x = 0$ and $x = L$.

(b) Write the Rodrigue's formula for Legendre polynomials. Using this formula, obtain the value of $P_3(x)$ and $P_4(x)$. 1+4

3. (a) What are symmetric and anti-symmetric matrices ? Separate the real and imaginary parts of a Hermitian matrix H by writing it as the sum of a real matrix A and i (the imaginary unit) times another real matrix B : 2+3

$$H = A + iB$$

What is the nature of the matrices A and B ?

(b) Obtain the eigen values for the real symmetric matrix : 2+3

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Determine the eigen vector for any one eigen value.

4. (a) Show that g^{ij} is a contravariant second rank tensor. 5

(b) Obtain the Taylor's series expansion of $\cos^2 z$ about $z = 0$. 5

5. Using the method of Residues (Jordan's lemma), show that : 10

$$\int_{-\infty}^{\infty} \frac{\cos x\, dx}{x^2+4} = \frac{\pi e^{-2}}{2}$$

6. (a) Consider a triangle P in the z-plane with vertices at i, $1-i$, $1+i$. Obtain the triangle P_0 which mapped P under the transformation $w = 3z + 4 - 2i$. What is the relation between P and P_0 ? Also calculate the area magnification. 5

(b) Obtain the Fourier transform of the function : 5

$$f(t)=\begin{cases} 0 \quad , & t<0 \\ e^{-at}, & t \geq 0,\ a>0 \end{cases}$$

7. Obtain the inverse Laplace transform of : 10

$$F(s)=\frac{s^2+2s+3}{(s+2)(s+1)^2}$$

8. (a) Define the symmetric group S_3 and give its geometrical illustration. 5

(b) Show that all $n \times n$ unitary matrices form a group. 5

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