No. of Printed Pages : 5

# MPH–008

## M. SC. (PHYSICS)

## (MSCPH)

## Term-End Examination

## December, 2024

## MPH-008 : QUANTUM MECHANICS–II

*Time : 2 Hours* *Maximum Marks : 50*

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***Note :*** *Answer any* ***five*** *questions. Symbols have their usual meanings. The marks for each question are indicated against it. You may use a calculator.*

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1. (a) Write the space translation operator for an infinitesimal translation $\Sigma x$ along the $x$-direction. Using the properties of space translation, show that $[\hat{p}_x, \hat{p}_y] = 0$. 5

(b) Show that the eigen values of the parity operator are ± 1. What are the even and odd parity eigen functions of the parity operator ? 5

2. Write the symmetric and anti-symmetric wave function for a system of two identical particles. Obtain the eigen value of the permutation operator for each of these states. State the symmetrization postulate. 2+6+2

3. Write down the direct product kets for a system of two angular momenta with $j_1 = j_2 = \frac{1}{2}$. For $\hat{J} = \hat{J}_1 + \hat{J}_2$, write the eigen kets of the total angular momentum $\hat{J}$ for this system. Using the direct product kets as a basis, determine the matrix $J_z$. 2+2+6

4. A two-dimensional infinite potential well is defined by : 2+8

$$V(x, y) = \begin{cases} 0, & \text{for } 0 \le x \le a; \ \ 0 \le y \le a \\ \infty, & \text{elsewhere} \end{cases}$$

Write the eigen functions for the two fold degenerate first excited state. Obtain the first order correction to the eigen energy of the first excited state for the perturbation :

$$H_1(x) = \begin{cases} \alpha, & \text{for } 0 \le x \le a/2 \\ 0^2, & \text{otherwise} \end{cases}$$

**Note :** The energy eigen functions for a one-dimensional infinite potential well of size L are given by :

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

5. (a) For a particle moving in a potential function $V(x)$, obtain the condition for a semi-classical approximation to be valid. 5

(b) Use the W-K-B approximation to derive the energy eigen value for a simple harmonic oscillator. 5

6. Consider a system described by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}(t)$ where the unperturbed Hamiltonian $\hat{H}_0$ has only two eigen kets $|\phi_1\rangle$ and $|\phi_2\rangle$ with the eigen energies $E_1$ and $E_2$ respectively. Derive the general equation for the time variation of the coefficients $a_1$ and $a_2$, where : 6+4

$$|\Psi(t)\rangle = a_1(t)\, e^{\frac{-iE_1 t}{\hbar}} |\phi_1\rangle + a_2(t)\, e^{\frac{-iE_2 t}{\hbar}} |\phi_2\rangle$$

What will be the equations for

$$V(t) = V_0\, e^{i\omega t} |\phi_1\rangle \langle\phi_2| + V_0\, e^{-i\omega t} |\phi_2\rangle \langle\phi_1| \,?$$

7. For elastic scattering by a potential $V(\vec{r})$ which has a finite range $a$, the asymptotic form of the wave function is : 10

$$\Psi(\vec{r}) = A\left[e^{ikz} + f_k(\theta, \phi)\frac{e^{ikr}}{r}\right]; r >> a$$

where $a$ is the range of the potential. Derive the expression for the differential scattering cross-section.

8. Derive the equation of continuity starting from the Klein-Gordon equation. 10

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