No. of Printed Pages : 3

MST–011

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

December, 2024

MST–011 : REAL ANALYSIS, CALCULUS AND GEOMETRY

Time : 2 Hours *Maximum Marks : 25*

Note : *Question No.* ***1*** *is compulsory. Attempt any* ***two*** *questions from Question Nos.* ***2*** *to* ***4****. Non-programmable scientific calculator is allowed. Symbols have their usual meanings.*

1. (a) Define appropriate function and then find the value of sgn (– 8.24). 2

   (b) Find the direction cosines of the normal to the plane $2x - 3y + 6z = 8$. 1

   (c) Without evaluating the integral $\int_0^\infty \frac{x^3}{(1+x)^9} dx$, find its value. 2

2. (a) Define inverse of a function. Show that exponential and logarithm functions are inverse of each other. Plot the graph of exponential and logarithm functions simultaneously on the same plot. 4

(b) Define each term of the triplet (Ω, A, μ). What is the name of this triplet ? Also, give *one* example to explain the meaning of each term of this triplet. 6

3. (a) Find the length/area/volume of the region formed by $2 \leq x \leq 7$ and $3 \leq y \leq 10$ using double integral. 2

(b) Evaluate $\int_0^\infty x^9 e^{-8x}\, dx$ using properties of beta and gamma functions. 1

(c) Prove that intersection of convex sets is also a convex set. 4

(d) Find the equation of the hyperplane in $\mathbf{R}^{10}$ in Cartesian form which passes through the point A (1, 2, 3, 4, 5, 6, 7, 8, 9, 0) perpendicular to the direction having direction ratio < 5, 4, 1, 7, – 3, 2, 1, 6, – 2, 3, >. 3

4. (a) If $f : [5, 15] \to \mathbf{R}$ be a function defined by $f(x) = 5x + 2, x \in [5, 15]$, show that f is Riemann integrable using definition 1 and definition 2. 5

(b) State and prove connection between convex set and convex function. 5

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