No. of Printed Pages : 4 **MST–012**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

December, 2024

MST–012 : PROBABILITY AND PROBABILITY DISTRIBUTIONS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions from question numbers **2** to **6**.*

(iii) Use of Scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. (a) State whether the following statement is True *or* False. Give reason in support of your answer : 5×2=10

"If (Ω, F, P) be a probability space, then every subset of Ω is an event and every event is a subset of Ω."

(b) Using axioms of probability theory, prove that $P(\phi) = 0$.

(c) Describe the hypergeometric distribution.

(d) If ϕ denotes CDF of standard normal distribution, then prove that :

$$P(a < z < b) = \Phi(b) - \Phi(a).$$

(e) Which one of the following is true and why ?

$$E\left(\frac{1}{X}\right) \leq \frac{1}{E(X)} \text{ or } E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$$

where $X > 0$.

2. (a) Suppose two friends Anjali and Prabhat trying to meet for a lunch say between 2 p.m. to 3 p.m. and they follow the following rules for this meeting :

   (i) Anyone can reach any time from 2 p.m. to 3 p.m. Assume that all possibilities of their reaching time from 2 p.m. to 3 p.m. are equally likely for both of them.

   (ii) Whoever of them reach first can wait for the other of meet only for 15 minutes. If within 15 minutes the other does not reach, he/she leaves the place and they will not meet.

   Find the probability of their meeting.

(b) If MGF of the random variable X is $\frac{1}{\sqrt{1-2t}}, t < \frac{1}{2}$, find MGF of Y = 10X.

(c) Explain the role of Jacobian in transformation through an example. 4+2+4

3. (a) In a city, 1000 births of babies take place each month out of which one in 100 births is of a twin. Find the probability that in the next month there will be 5 twins.

(b) The power ball game is a game where, you have to choose 5 white balls numbered 1 to 55 and 1 red ball numbered 1 to 42. In this game there are various prizes like, you choose (i) all the five winning white balls and power ball, where winning red ball is known as power ball (ii) only five winning white balls (iii) only four winning white balls, and so on. Find the probability of only selecting 3 winning white balls.

(c) If X ~ Bern (p), then discuss the behaviour of variance with respect to its parameter.

3+4+3

4. (a) Using definition of CDF, obtain it from PDF of Cauchy distribution. Using obtained CDF, find the quantile function of the same distribution.

(b) Suppose that temperature of a particular city in the month of February is normally distributed with mean 24°C and standard deviation 5°C. Find the probability that temperature of the city on a day of the month of February is (i) less than 20°C, (ii) more than 26°C, (iii) between 22°C and 27°C. 4+6

5. (a) It is known that sum of 1000 numbers is 100000 and sum of their squares is 10025. On the basis of this information, find an upper bound on how many numbers out of these 1000 numbers are ≥ 110.

(b) Let (Ω, F, P) be a probability space and we have a random variable X and a sequence of random variables X_1, X_2, X_3, ... such that $X_n \xrightarrow{\text{m.s.}} X$. Does it always imply that $X_n \xrightarrow{\text{a.s.}} X$. If your answer is yes, then prove your claim and if your answer is no, then give a counter-example. 5+5

6. State and prove strong law of large numbers. 10

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