M. SC. (APPLIED STATISTICS) (MSCAST)

Term-End Examination December, 2024

MST-014: STATISTICAL QUALITY CONTROL AND TIME SERIES ANALYSIS

Time: 3 Hours Maximum Marks: 50

Note: (i) Question No. 1 is compulsory.

- (ii) Attempt any four questions from the remaining Question Nos. 2 to 6.
- (iii) Use of scientific calculator (non-programmable) is allowed.
- (iv) Symbols have their usual meanings.
- 1. State whether the following statements are True or False. Give reasons in support of your answers: $5\times2=10$
 - (a) If a component has the following hazard rate:

$$\lambda(t) = 0.5t, \quad t \ge 0$$

then the reliability of the component will be $e^{-t^2/4}$.

- (b) The average total inspection (ATI) is equal to the average sample number (ASN), when the rejected lot is 100% inspected.
- (c) For controlling the number of defects in a process, the *p*-chart is used.
- (d) An auto-regressive AR(2) model:

$$X_t = 0.80 X_{t-1} - 0.60 X_{t-2} + a_t$$

is stationary.

- (e) The C_{pk} index is better than the C_p index because it considers the shifting of variance.
- 2. A company produces dry cells. The cells will be considered satisfactory, if their life is at least 25 hours. To test for the process to be under statistical control, a sample of 4 cells was drawn on 7 consecutive days and measured their lives. The results are as follows:

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
24	16	20	26	20	21	26
20	18	25	25	22	40	24
25	15	28	26	24	20	20
20	22	30	24	26	20	25

(i) Estimate the process mean and variability.

- (ii) Determine the center line and control limits of the control chart for controlling the process mean.
- (iii) Plot the control chart for mean. Comment on the status of the process in respect of the process mean.
- (iv) If necessary, complete revised control limits.

(Given : $d_2 = 2.059$, $A_2 = 0.729$)

- 3. A manufacturer of men's jeans purchases zippers in lots of 1000. A single sampling plan with sample size 15 is used and it is decided that if in a lot, more than 2 zippers are found defective, the lot will be rejected. It is also decided that AQL = 0.4 and LTPD = 0.12. If there are 5% defective zippers in each lot, calculate:
 - (i) Probability of accepting the lot
 - (ii) Producer's risk
 - (iii) Consumer's risk

- (iv) AOQ under acceptance and rectifying sampling plan
- (v) ASN

Given: that:

$$\sum_{x=0}^{2} {}^{15}C_x(p)^x (1-p)^{15-x} = \begin{cases} 0.9639 & \text{for } p = 0.04\\ 0.9797 & \text{for } p = 0.05\\ 0.7346 & \text{for } p = 0.12 \end{cases}$$

4. The following data, give the average quarterly prices of a commodity for three years:

Year	2015	2016	2017
Quarter			
I	13	14	15
II	12	16	15
III	15	18	18
IV	15	17	19

- (i) Compute the seasonal indices using ratio to moving average method.
- (ii) Do seasonal forces significantly influence the price of the commodity. 8+2
- 5. The failure density function of life of a component of a system (T) is given by:

$$f(t) = 0.2e^{-0.2t}; t > 0$$

Calculate:

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- (i) Reliability of the system
- (ii) Hazard rate
- (iii) Mean time to failure
- (iv) Median of the random variable 'T'
- (v) What is life of the component if the reliability of 0.90 is desired?
- 6. (a) Consider a time series model:

$$y_t = 10 + 0.2y_{t-1} + \epsilon_t$$

where $\in_t \sim N(0, 1)$.

- (i) Is this a stationary time series?
- (ii) What are the mean and variance of the time series?
- (b) Define redundancy.

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