

**M. SC. (APPLIED STATISTICS)
(MSCAST)**

**Term-End Examination
December, 2024**

MST-018 : MULTIVARIATE ANALYSIS

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining Question Nos. 2 to 6.*

(iii) *Use of scientific calculator (non-programmable) is allowed.*

(iv) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answers :

$$2 \times 5 = 10$$

- (a) Multivariate normal distribution is an analogue of univariate normal distribution.

- (b) The first and second principal components in principal component analysis depend on each other.
- (c) If :

$$A = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 8 & -1 \\ 2 & -1 & 4 \end{pmatrix}$$

and
$$B = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 3 & 0 \\ -2 & 0 & 6 \end{pmatrix},$$

then the trace of matrix A is greater than trace of matrix B.

- (d) If the variance-covariance matrix Σ of a random vector $\tilde{X}$ is partitioned as :

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

then $\Sigma_{21} = \Sigma_{12}$.

- (e) Canonical correlation is the maximum correlation between the variates and linear combination of the variates.

2. (a) Let $\tilde{X}_{p \times 1} \sim N_p(\tilde{\mu}, \tilde{\Sigma})$. Also let $\tilde{X}$, $\tilde{\mu}$ and $\tilde{\Sigma}$ be

partitioned as :

$$\tilde{X}_{p \times 1} = \begin{pmatrix} \tilde{X}_{k \times 1}^{(1)} \\ \tilde{X}_{(p-k) \times 1}^{(2)} \end{pmatrix}, \quad \tilde{\mu}_{p \times 1} = \begin{pmatrix} \tilde{\mu}_{k \times 1}^{(1)} \\ \tilde{\mu}_{(p-k) \times 1}^{(2)} \end{pmatrix} \text{ and}$$

$$\tilde{\Sigma}_{p \times p} = \begin{pmatrix} \Sigma_{11} \ k \times k & \Sigma_{12} \ k \times (p-k) \\ \Sigma_{21} \ (p-k) \times k & \Sigma_{22} \ (p-k) \times (p-k) \end{pmatrix}.$$

Then prove that the sub-vectors $\tilde{X}^{(1)}$ and

$\tilde{X}^{(2)}$ are independently normally

distributed if and only if $\Sigma_{12} = 0$, i.e.,

$\text{Cov}\left(\tilde{X}_{k \times 1}, \tilde{X}_{(p-k) \times 1}^{(2)}\right) = 0$, where 0 is the

null matrix. 5

(b) Let $\tilde{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ has the following joint

density function :

$$f(x_1, x_2) = \begin{cases} 2(x_1 + x_2 - 4x_1^3 x_2), & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then find the mean vector of $\tilde{X}$ and the

diagonal elements of the variance-covariance matrix of $\tilde{X}$. 5

3. (a) Let $\tilde{X}$ be a 3-dimensional vector with

dispersion matrix $\Sigma = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & 2 \end{pmatrix}$. Then

determine the first principal component and the proportion of the total variability that it explains. 5

- (b) If $\tilde{X}_{p \times 1} \sim N_p(\tilde{\mu}, \Sigma)$, where Σ is a diagonal matrix, then the components of

$$\tilde{X}_{p \times 1} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_p \end{pmatrix} \text{ are independently normally}$$

distributed as $X_i \overset{\text{Ind.}}{\sim} N_1(\mu_i, \sigma_{ii})$ for $i = 1, 2, \dots, p$.

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4. (a) If $\Sigma = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the variance-

covariance matrix of a random vector $\tilde{X}$,

then obtain the corresponding correlation coefficient's matrix. 5

- (b) If $\tilde{X} \sim N_p(\tilde{\mu}, \tilde{\Sigma})$, then prove that :

$$\tilde{Y}_{q \times 1} = C_{q \times p} \tilde{X}_{p \times 1} + \tilde{V}_{p \times 1} \sim N_q\left(C_{\tilde{\mu}} + \tilde{V}, C \tilde{\Sigma} C'\right),$$

where C is any $q \times p$ matrix of rank $q (\leq p)$ of constant elements and $\tilde{V}_{p \times 1}$ is a p component vector of constants. 5

5. (a) Define Wishart distribution. If $A_1 \sim \omega(n_1, p, \Sigma)$ and $A_2 \sim \omega(n_2, p, \Sigma)$, where A_2 is independent of A_1 , then prove that : 5

$$A_1 + A_2 \sim \omega(n_1 + n_2, p, \Sigma).$$

- (b) Define sample discriminant function. Write the stepwise classification procedure using sample discriminant function. 5

6. Obtain and draw the dendrogram for the following distance matrix using single linkage method : 10

$$D = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \left(\begin{array}{ccccc} 0 & & & & \\ 7 & 0 & & & \\ 2 & 5 & 0 & & \\ 4 & 4 & 3 & 7 & 0 \\ 9 & 8 & 1 & 6 & 0 \end{array} \right) \end{matrix}$$

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