No. of Printed Pages : 5 **MST–021**

M. SC. (APPLIED STATISTICS) (MSCAST)

Term-End Examination

December, 2024

MST-021 : CLASSICAL AND BAYESIAN INFERENCE

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions from the remaining Question Nos. **2** to **6**.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. State whether the following statements are True or False. Give reasons in support of your answer : 5×2=10

   (a) If the form of the population is not known and data are in ordinal form, then we apply Wilcoxon signed rank test for testing hypothesis about average.

(b) In Bayesian approach, the prior distribution is not known.

(c) The Neyman-Pearson lemma provides the most powerful test of size α for testing simple hypothesis against simple alternative hypothesis.

(d) The Cramer-Rao inequality with regard to the variance of an estimator provides upper bound on the variance.

(e) The non-parametric tests are more powerful than the parametric tests.

2. A radar system uses radio waves to detect aircraft. The system receives a signal and based on the received signal, it needs to decide whether an aircraft is present or not. If X denotes the received signal, then :

$$X = \theta + W = \begin{cases} X = W \quad , & \text{if no aircraft is present} \\ X = 1 + W, & \text{if an aircraft is present} \end{cases}$$

where $W \sim N\left(0, \frac{1}{9}\right)$.

for testing the hypothesis :

H_0 : No aircraft is present, i.e., $\theta = 0$

H_1 : An aircraft is present, i.e., $\theta = 1$

derive the test at $\alpha = 0.05$. 10

3. A company has selected 10 employees randomly to assess the effectiveness of the training programme.

The company noted the scores (out of 100) obtained by the employees before and after the training which are given as follows : 10

S. No.	**Before Training**	**After Training**
1	60	68
2	62	70
3	67	80
4	64	74
5	66	66
6	63	72
7	69	84
8	63	60
9	60	65
10	62	90

To, test whether the training programme has improved the efficiency of the employee, give the answers of the following :

(i) Are the samples paired or independent ?

(ii) State the null and alternative hypotheses.

(iii) Write the assumptions of Wilcoxon matched-pair signed-rank test.

(iv) Conduct the test at 5% level of significance and conclude the result.

4. (a) State Lehmann-Scheffe theorem. 2

(b) If $X_1, X_2, \ldots, X_n$ is a random sample taken from a Poisson distribution, whose p.m.f. is given as :

$$P[X = x] = \frac{e^{-\lambda}\lambda^{x}}{x!}; \; x = 0, 1, \ldots \; \lambda > 0$$

(i) Show that statistic $\sum_{i=1}^{n} X_i$ is sufficient and complete.

(ii) Show that $\frac{1}{n}\sum_{i=1}^{n} X_i$ is the uniformly minimum variance unbiased estimator. 4+4

5. A faculty member of a university receives a number of e-mails. If X represents the number of spam e-mails out of received n e-mails and

follow Binomial distribution with parameter (n, θ), where θ is the probability of getting a spam e-mail, then find the posterior distribution of θ considering the following beta distribution as prior :

$$f(\theta) = \frac{1}{\beta(a, b)} \theta^{a-1}(1-\theta)^{b-1}, 0 \leq \theta \leq 1, a, b > 0$$

Also find the posterior mean of θ. 10

6. (a) A train is expected to arrive at a station at 8 : 00 a.m. However, it has been observed that it reaches station between 7 : 55 a.m. to 8 : 05 a.m. and the times are uniformally distributed between the 7 : 55 a.m. to 8 : 05 a.m. interval. Using the following U(0, 1) random numbers, simulate time for arrival on ten days : 6

   0.579, 0.052, 0.312, 0.307, 0.645, 0.945, 0.645, 0.956, 0.394, 0.110

   (b) Differentiate between parametric and non-parametric tests. 4

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