

Ph. D. IN STATISTICS (PHDSTAT)

Term-End Examination

December, 2024

RST-003 : ADVANCED SAMPLE SURVEYS

Time : 3 Hours

Maximum Marks : 100

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from Question Nos. 2 to 7.*

(iii) *Only non-programmable/scientific calculator is allowed.*

(iv) *Symbols have their usual meanings.*

1. (a) State whether the following statements are true or false. Give reasons in support of your answers : $4 \times 2 = 8$

(i) A simple random sample with replacement of size 16 is drawn from a population with 50 members. The SE of sample mean, if the population variance is known to be 25, will be 1.25.

- (ii) From a population of size 50, a random sample of size 10 is drawn. The MSE of ratio estimator $\bar{y}_R$ of $\bar{Y}$ is calculated for $C_y = 2$ and $C_x = 0.50$, which comes to be $0.08\bar{Y}^2$. The value of the correlation coefficient ρ_{yx} will be 1.
- (iii) The inadequate scrutiny of basic data is a type of sampling error.
- (iv) If the equal size strata are having stratum mean squares in the ratio 1 : 2 : 3, then the sample drawn from each stratum under optimum allocation will be in the ratio 1 : 1 : 1.
- (b) Differentiate between the following :
- $4 \times 3 = 12$
- (i) Stratified and Cluster sampling
 - (ii) Ordered and Unordered estimators
 - (iii) One-stage and Two-stage sampling
 - (iv) Variance and Mean squared error

2. Consider a form of product estimator :

$$\bar{y}_{pg} = \frac{\bar{y}(x + \alpha)}{(X + \alpha)}$$

for estimating the population mean $\bar{Y}$, where α is a constant. 15+5

- (i) Derive the approximate bias and mean squared error of $\bar{y}_{pg}$.
- (ii) Determine the value of α , for which the mean squared error is minimum.
3. Define two-stage sampling and give its advantages. Write an unbiased estimator of population mean and derive its sampling variance. 20
4. A simple random sample without replacement of villages in each stratum of a district was selected and the number of apple orchards for various strata are given below :

Stratum	Total number	Number of villages selected	Number of orchards in the selected villages
A	75	10	2, 5, 1, 9, 6, 7, 4, 7, 5, 3
B	57	9	21, 11, 7, 5, 6, 19, 5, 24, 30
C	43	6	3, 10, 4, 11, 18, 19
D	25	5	30, 42, 20, 38, 29

Estimate the number of orchards in the district. Determine whether there is any gain due to stratification over simple random sampling. 20

5. Show that the Hurwitz and Thompson estimator is an unbiased estimator of population mean. Also, derive the expression for its variance. 20
6. Suppose there are two strata and equal size of samples are drawn from both the strata, i.e. $n_1 = n_2$. Another option to draw the samples is optimum allocation, i.e. $n_{1\text{opt}}$ and $n_{2\text{opt}}$. Let V_{eq} and V_{opt} denote the variances under equal and optimum allocation respectively, then assuming N_1 and N_2 are large, show that :

$$\frac{V_{\text{eq}} - V_{\text{opt}}}{V_{\text{opt}}} = \left(\frac{\delta - 1}{\delta + 1} \right)^2$$

$$\text{where } \delta = \frac{n_{1\text{opt}}}{n_{2\text{opt}}}. \quad 20$$

7. Write short notes on any *two* of the following : 10+10
 - (a) Successive Sampling
 - (b) Post-stratification
 - (c) Double Sampling

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