No. of Printed Pages : 5

RST–005

Ph. D. IN STATISTICS (PHDSTAT)

Term-End Examination

December, 2024

RST–005 : STATISTICAL INFERENCE IN LIFE TESTING

Time : 3 Hours *Maximum Marks : 100*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions from Q. Nos.* ***2*** *to* ***7****.*

(iii) Use of scientific non-programmable calculator is allowed.

(iv) Symbols have their usual meanings.

---

1. (a) State whether the following statements are True *or* False. Give reasons in support of your answers : 2×6=12

(i) The Rao-Blackwell theorem enables us to obtain minimum variance unbiased estimator through complete statistic.

(ii) The Cramer-Rao bound on the variance of an estimator T of parameter θ is :

$$\text{Var (T)} \geq \frac{(\theta')^2}{n\text{E}\left[\frac{\partial^2}{\partial\theta^2}\log f(x)\right]}$$

(iii) The form of Entropy Loss Function is :

$$\text{L}(\Delta) \propto \Delta^{p+1} - \log\Delta^p - 1$$

where $\Delta = \frac{\hat{\theta}}{\theta}$.

(iv) If we do not get the information about the failure of all units of an experiment, then obtained data is called complete data.

(v) The numbers generated from a deterministic process is called random numbers.

(vi) The reliability function of an exponential failure distribution with parameter θ is $1 - e^{-\theta t}$.

(b) A researcher observed that the inter-arrival time of trucks arrive in a

warehouse follows exp (θ) with $\theta = 0.2$. Generate inter-arrival times of six trucks using the following $U(0,1)$ random numbers :

8

$u_1 = 0.16$, $u_2 = 0.30$, $u_3 = 0.75$, $u_4 = 0.51$, $u_5 = 0.42$, $u_6 = 0.10$.

2. A faculty member of a university receives a number of e-mails. If X represents the number of spam e-mails in m e-mails and follows binomial distribution with parameter (m, θ), where θ is the probability of getting spam e-mails, then find the posterior distribution of θ considering the following beta distribution as prior :

$$f(\theta) = \frac{1}{\beta(a,b)} \theta^{a-1} (1-\theta)^{b-1}, \quad 0 \le \theta \le 1, a, b > 0.$$

Also find the Bayes estimate of θ under SELF.

20

3. Explain various techniques for generating random numbers. Also write their drawbacks.

20

4. (a) Describe Gibbs sampling. 10

(b) Solve the following definite integration using Monte-Carlo integration : 10

$$\int_0^2 e^{\frac{-x^2}{2}} dx$$

using random numbers given in 1 (b).

5. (a) For the following failure density function : 15

$$f(x) = \alpha\lambda^{\alpha} x^{\alpha-1} e^{(\lambda x)^{\alpha}} \exp\left(1 - e^{(\lambda x)^{\alpha}}\right),\ x > 0, \alpha, \lambda > 0$$

Find :

(i) Reliability function,

(ii) Cumulative failure distribution function

(iii) Hazard function

(b) If a system has n components which are connected in series, then derive the reliability expression : 5

$$\mathrm{R}(t) = \prod_{i=1}^{n} \mathrm{R}_i$$

of the system.

6. Find the Cramer-Rao lower bound for variance of estimator $\overline{X}$ of the parameter θ of the following distribution : 20

$$f(x) = \frac{1}{\theta} e^{-x/\theta}; \quad x > 0, \theta > 0$$

Also show that $\overline{X}$ is UMVUE for θ.

7. Write short notes on any *two* of the following : 10+10

(i) Time and failure censoring schemes

(ii) Metropolis algorithm

(iii) Various loss functions

× × × × × × ×