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M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2025

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours Maximum Marks :25

Weightage : 70%

Note : *(i) There are five question in this paper.*

*(ii) The **fifth** question is compulsory.*

*(iii) Do any **three** questions from Q. Nos. **1** to **4**.*

1. (a) Let T be a linear operator from $\mathbf{R}^2$ to itself whose matrix with respect to the ordered basis $B = \left\{ \begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ is $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. Find its matrix with respect to the ordered basis $B' = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$. 3

(b) Check whether the system of equations :

$$x + 2y = 3$$

$$x - y = 1$$

$$2x + y = 2$$

is consistent. If the system is consistent, use Guassian elimination to solve it. If the system is not consistent, find the least squares solution to the system of equations. 2

2. Find the singular value decomposition of the matrix : 5

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

3. Find the Jordan canonical form of : 5

$$A = \begin{bmatrix} 5 & -4 \\ 1 & 1 \end{bmatrix}.$$

Hence solve the system of differential equations :

$$\frac{dy(t)}{dt} = Ay(t), \text{ where } y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

4. (a) Find the square root of the matrix : 3

$$S = \begin{bmatrix} 5 & 0 & -4 \\ 0 & 5 & 0 \\ -4 & 0 & 5 \end{bmatrix}.$$

(b) Define a nilpotency index of a nilpotent matrix. Find the nilpotency index of the matrix : 2

$$\begin{bmatrix} -4 & 1 & 1 \\ -10 & 2 & 3 \\ -8 & 2 & 2 \end{bmatrix}.$$

5. Which of the following statements are true and which are false ? Justify your answer with a short proof or a counter example. Marks will be given only for proper justification : 5

(a) The Jordan canonical form of any square matrix is unique.

(b) There is no matrix with minimal polynomial $(x - 1)(x - 2)(x - 3)$.

(c) If all the entries of a symmetric matrix are positive its eigen values are positive.

(d) Every square matrix is normal.

(e) The QR decomposition of any matrix is unique.

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