No. of Printed Pages : 4 **MMT–005**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2025

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours Maximum Marks : 25

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***three*** *questions from Question Nos.* ***2*** *to* ***5****.*

(iii) Use of calculator is not allowed.

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1. State, giving reasons whether the following statements are True *or* False : 2×5=10

(a) The set $\left\{z \in \mathbf{C} : 0 < \arg z < \frac{\pi}{4}\right\}$ is a domain.

(b) The function $f(z) = |z|^2$ is nowhere differentiable.

(c) The identity :

$$\log\left(\frac{z_1}{z_2}\right) = \log z_1 - \log z_2$$

holds for every pair of non-zero complex numbers z_1 and z_2.

(d) $\int_C \frac{1}{z^2 + 5z + 6} dz = 0$, where C denotes the positive oriented circle $|z| = 1$.

(e) The function $f(z) = \cos z$ is conformal everywhere.

2\. (a) Prove that the function : 3

$$u(x, y) = y^3 - 3x^2 y$$

is harmonic. Find its harmonic conjugate.

(b) Consider the region $R = \{z : |z| \le 3\}$. If $f(z) = 3 - z$ in R, then find a point in R, where $|f(z)|$ attains its maximum value. 2

3. (a) Find the value of log $(-1 - i)$. 2

(b) (i) Prove that the Mobius transformation $w = \frac{2z-1}{2-z}$ maps unit disc to itself. $1\frac{1}{2}$

(ii) Prove that if the sequence (z_n) converges to z, then the sequence $(|z_n|)$ converges to $|z|$. $1\frac{1}{2}$

4. (a) Find the Laurent's series expansion in powers of z for the function : 3

$$f(z) = \frac{1}{z^4 + 3z^2 + 2}$$

in the region $1 < |z| < \sqrt{2}$. Also write its principal part at $z = 0$.

(b) Without evaluating the integral, show that : 2

$$\left|\int_C \frac{1}{z^4 - 5z^2 + 4}\, dz\right| \le \frac{3\pi}{40}$$

where C is upper half of the circle $|z| = 3$.

5. (a) Use Cauchy Residue theorem to evaluate : 3

$$\int_0^{\pi} \frac{a\, d\theta}{a^2 + \sin^2 \theta}, \qquad a > 0$$

(b) Find the radius of convergence of the power series : 2

$$\sum_{n=1}^{\infty} \left[5 + 2(-1)^n\right]^n z^n$$

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