No. of Printed Pages : 5 **MMT–006**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2025

MMT-006 : FUNCTIONAL ANALYSIS

Time : 2 Hours *Maximum Marks : 50*

Weightage : 70%

Note : *(i) Question No. **1** is compulsory.*

*(ii) Answer any **four** questions from the remaining.*

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1. State True *or* False, with justification, for each statement : 5×2=10

(a) $\mathbf{R}^2$ with any norm is complete.

(b) 0 is not an eigen value of any non-zero compact operator on an infinite-dimensional Banach space.

(c) If M is a closed subspace of a Hilbert space, there is a subspace N such that $M = N^{\perp}$.

(d) Every linear subspace of X × X is the graph of a linear map.

(e) X' separates points of X for any normed space X.

2. (a) Let M be a closed subspace of a Banach space X. Prove that there is a closed subspace N such that $X = M \oplus N$ if and only if there is a bounded projection P with R(P) = M. 4

(b) Show that a compact self-adjoint operator has an eigen value. 3

(c) Prove that the dual space of C_0 is l^1. 3

3. (a) If $\{e_i\}$ is an orthnormal set in a Hilbert space H such that

$$\|x\|^2 = \Sigma |<x, e_i>|^2, \forall x \in \mathrm{H},$$

then prove that $\{e_i\}$ is an orthnormal basis. 3

(b) Show that for any bounded linear operator A on a Banach space X, the series $\sum_0^\infty \frac{\mathrm{A}^n}{n!}$ converges to a bounded invertible operator. 2+2

(c) Prove that a normed space in which every absolutely convergent series is convergent is complete. 3

4. (a) If M is a substance of a normed space X, show that M′ is linearly isometric to $\mathrm{X}'/\mathrm{M}^{\perp}$. 4

(b) Define a positive operator. Let A be bounded self-adjoint operators on a Hilbert space H. Show that : 2+4

(i) A^2 is a positive operator.

(ii) If A is a positive operator, then I + A is invertible.

5. (a) Let X be a normed space. Let : 3

$$B(x, s) = \{y \in X : \|x - y\| > s\}$$

Show that :

$$\overline{B(x, s)} = \{y \in X : \|x - y| \leq s\}$$

(b) State open mapping theorem. Give an example to show that the open mapping theorem may not hold if the normed spaces are not Banach spaces. 4

(c) Calculate the adjoint of the operator S defined on l^2 by $S(x) = (x(2), x(3), ...)$.

3

6. (a) If A is a bounded operator on an infinite dimensional Banach space X such that $\|Ax\| \geq C\|x\|$ for all $x \in X$ and some $C > 0$, prove that A cannot be compact. 4

(b) Define an approximate eigen value and give an example. 1+2

(c) Prove that the space $C^1[0, 1]$ is complete with the norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$. 3

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