No. of Printed Pages : 8 **MMT–007**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2025

MMT–007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

(Weightage : 50%)

Note *: (i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions out of question nos. 2 to 7.*

(iii) Use of scientific (non-programmable) calculate is allowed.

---

1. State whether the following statements are True *or* False. Justify your answer with the help of a short proof or a counter-example.

No marks are awarded for a question without justification : 2×5=10

(a) The non-trivial solution of the boundary value problem $y'' + \lambda y = 0$ with boundary condition $y(0) = 0$ and $y(\pi) = 0$ is $y(x) = a_1 \sin x$, or $a_2 \sin 2x$, or $a_3 \sin 3x$, or ....; and solution vanishes at the end points 0 and π of interval $[0, \pi]$.

(b) IVP $y' = e^{-x^2} y^2 \sin x$ does not satisfy the Lipchitz condition on the region R, where R is the strip $0 \le y \le 2$ in the *xy*-plane.

(c) $y = c_1 x^2 + c_2 x^2 \ln x$ is the general solution to the equation $x^3 y''' - 2xy' + 4y = 0$, on the interval $]0, \infty[$.

(d) $\frac{1}{4}\left[-xe^x + \frac{x^2}{2}e^x + \frac{1}{2}e^x + \frac{1}{4}e^x\right]$ is the particular integral of $(D-1)^2(D+1)^2 y = e^x$.

(e) The series $\sum_{n=0}^{\infty} n^n x^n = 1 + x + 2^2x^2 + 3^3x^3 + \ldots$ converges for any non-zero x.

2. (a) Determine the series solution about the origin of the equation : 5

$$9x(1-x)y'' - 12y' + 4y = 0$$

(b) Using Rodrigue's formula, show that : 2

$$\int_{-1}^{1} x^n P_n(x)\, dx = \frac{2^{n+1}(n!)^2}{(2_{n+1})!}$$

(b) Find the Legendre series for : 3

$$f(x) = \begin{cases} -1, & -1 \le x \le 0 \\ 1, & 0 < x \le 1 \end{cases}$$

3. (a) Express $f(x) = e^x$ in a Hermite series and use the result to deduce the value of the integral $\int_{-\infty}^{\infty} e^{-x^2+x} H_n(x)\,dx$. 3

(b) Show that : 3

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\cos\theta)\,d\theta.$$

(c) Construct Green's function for the boundary value problem $\frac{d^2y}{dx^2} - y = x$; $y(0) = y(1) = 0$. 4

4. (a) Use the Laplace transform method to solve the system of equations $x'(t) = x - 2y$ and $y'(t) = 5x - y$, subject to initial conditions $x(0) = 1, y(0) = 1$. 3

(b) Find the Fourier sine transform of : 3

$$f(x) = \begin{cases} x & , \quad 0 < x < 1 \\ (2-x), & \quad 1 < x < 2 \\ 0 & , \quad x > 2 \end{cases}$$

(c) Find the number of terms that are to be retained if an accuracy of 10^{-10} is required in solving the initial value problem $\frac{dy}{dx} = x + y,\ y(0) = 1,\ x \in]\,0, 1\,[$ by Taylor's series. 4

5. (a) Find the approximate value of $y(0.8)$ for the initial value problem $y' = \sqrt{x+y},\ y(0.4) = 0.41$ using the second order Runge-Kutta method with $h = 0.2$. 3

(b) Determine the constants a_1, b_1 and b_2 in the explicit multistep method $y_{i+1} = a_1 y_i + h\left(b_1 y'_i + b_2 y'_{i-1}\right)$. Also find the truncation error and the order of the method. 4

(c) Solve the boundary value problem $y'' = xy + 1,\ y(0) + y'(0) = 1,\quad y(1) = 1$ using the second order finite difference method with $h = \frac{1}{2}$. 3

6. (a) Obtain approximate value of $y(0.4)$ for the initial value problem $y' = -2xy^2$, $y(0) = 1$ using predictor-corrector method, where : 5

$$\text{Predictor}\quad (\text{P}) \Rightarrow y_{i+1} = y_i + \frac{h}{2}\left[3y_i' - y_{i-1}'\right]$$

$$\text{Corrector}\ (\text{C}) \Rightarrow y_{i+1} = y_i + \frac{h}{2}\left[y_{i+1}' + y_i'\right]$$

with step length $h = 0.2$. Perform two corrector iterations per step and use $y = \dfrac{1}{1+x^2}$ to obtain the starting value.

(b) Using modified Euler's method find $y(0.1), y(0.2)$ given that $\frac{dy}{dx} = (x^2 + y^2)$, $y(0) = 1$. 5

7. (a) Find the solution of $\nabla^2 u = 0$ in R subject to the boundary conditions $u(x, y) = x^2 + y^2$ on τ, where R is the square $0 \le x \le 1, 0 \le y \le 1$ subject to $u = \frac{1}{12}(x^4 + y^4)$ on the lines $x = 1, y = 0, y = 1$ and $12u + \frac{\partial u}{\partial x} = x^4 + y^4 + \frac{x^3}{3}$ on $x = 0$ using the five point formula. Assume uniform step length $h = \frac{1}{2}$ along the axes. Use central difference approximation in the boundary condition. 4

(b) Use Schmidt method to find the solution of the heat conduction equation subject to the given initial and boundary conditions : 3

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1, u(x, 0) = \sin(\pi x),$$

$$0 \le x \le 1, u(0, t) = 0 = u(1, t)$$

(c) Apply Picard's iteration method to solve the initial value problem : 3

$$\frac{dy}{dx} = 1 + xy, \; y(2) = 0$$

to obtain y_1 and y_2.

× × × × ×