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M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2025

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 Hours *Maximum Marks : 100*

Weightage : 50%

Note : *(i) Question No. 8 is compulsory. Attempt any* ***six*** *questions from question nos.* ***1*** *to* ***7****.*

(ii) Use of scientific (non-programmable) calculator is allowed.

(iii) Symbols have their usual meanings.

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1. (a) In an investigation related to length of centrum (X_1) and width of the centrum

taken from interior (X_2) of 12 fish specimens belonging to families Serranidae (X_1) and Carengidae (X_2) were measured in mm. The mean and the sample covariance matrix were found to be :

$$\overline{X} = [7.375 \quad 6.808], \ S = \begin{bmatrix} 1.1919 & 0.5877 \\ 0.5877 & 0.3741 \end{bmatrix}.$$

In order to test the null hypothesis that observations came from a population with mean vector $\mu_0 = \begin{bmatrix} 8.94 \\ 6.76 \end{bmatrix}$, apply a suitable test statistic. You may consider $\alpha = 0.01$ for the test.

[Given that : [$F_{2,10,.0.01} = 7.56$]. 9

(b) If the random variables X and Y have the joint p.d.f. $f_{X,Y}(x, y) = e^{-x-y}$, $x > 0$, $y > 0$, find the following : 6

(i) $P[X \leq x]$

(ii) $P[Y \le y]$

(iii) $P[X < Y]$

(iv) $P[X + Y \le 3]$

2. (a) Consider the queuing model (M/M/C) : $(\infty/F/F_0)$, where there are C parallel service channels. The arrival rate is λ and the service rate per service channel is μ. Find the steady-state solution of the model.

Let in a queuing system, given that $\lambda = 12$ customers per hours, $\mu = 10$ per hour, C = 2. What will be the probability that an arrival has to wait ? 8

(b) Suppose that the two-dimensional continuous random variable (X, Y) has the joint p.d.f. : 7

$$f(x, y) = x^2 + \frac{xy}{3},\ 0 \le x \le 1,\ 0 \le y \le 2.$$

Then :

(i) find the marginal p.d.f.'s of X and Y.

(ii) find the conditional p.d.f.'s $g\ (x/y)$, and $h(y/x)$.

(iii) check whether $g(x/y)$ is a proper p.d.f.

3. (a) Let $\{X_n; n \geq 0\}$ be a Markov chain with four states 0, 1, 2 and 3 and the following transition probability matrix : 9

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1/6 & 1/3 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/6 & 1/3 & 1/2 & 0 \\ 0 & 1/6 & 1/3 & 1/2 \end{bmatrix} \end{array}$$

(i) Find the probability $P[X_2 = 2 \mid X_1 = 1]$ and $P[X_2 = 2, X_1 = 1, X_0 = 2]$, given that the initial distribution is $P[X_0 = i] = \frac{1}{4}$ for $i = 0, 1, 2, 3$ ....

(ii) Show that states 0, 1, 2 are persistent but state 3 is transient.

(b) Let X and Y have the following joint p.d.f. : 6

X \ Y	1	2	3
1	0.1	0.1	0
2	0.1	0.2	0.3
3	0.1	0.1	0

(i) Using the above table, find E(X + Y), E(XY), V(X + Y), E(X/Y).

(ii) Find the p.d.f. of X and Y and determine E(X), E(Y), V(X) and V(Y).

(iii) Using the results obtained in (i) and (ii), compare E(X + Y) and E(X) + E(Y), E(XY) and E(X)E(Y).

(iv) Are X and Y is dependent random variables ?

4. (a) In a factory, a certain type of product is produced by three machines M_1, M_2 and M_3. The daily production on these machines are :

M_1 : 3000 units, M_2 : 4500 units, M_3 : 2500 units.

From the past experience it is known that 1% of the production by M_1 are defective, 1.4% of the production by M_2 are defective and 2.2% of the production by M_3 are defective. An item is drawn at random from the day's production run and is found to be defective. Find the probabilities that it comes from the output of (i) M_1, (ii) M_2, (iii) M_3. 7

(b) The variates $X' = [X_1, X_2, X_3]$ and $Y' = [Y_1, Y_2, Y_3]$ are distributed independently according to trivariate normal populations with respective parameters :

$$\mu_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \Sigma_1 = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

and $\mu_2 = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \Sigma_2 = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

respectively.

Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ and

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

be non-singular matrices, then what are the distributions of the variates AX and BY ? 8

5. (a) Mention in brief the process of testing the equality of two mean vectors when two data matrices come from $N_p(\mu_1, \Sigma_1)$ and $N_p(\mu_2, \Sigma_2)$, where $\Sigma_1 = \Sigma_2 = \Sigma$(say). The two mean vectors $\overline{X}_0$ and $\overline{X}_1$ for the two data matrices are given as : 8

$$\overline{X}_0 = \begin{bmatrix} 5.15 \\ 0.85 \end{bmatrix}, \overline{X}_1 = \begin{bmatrix} 4.821 \\ 0.500 \end{bmatrix}$$

and respective sample covariance matrices are :

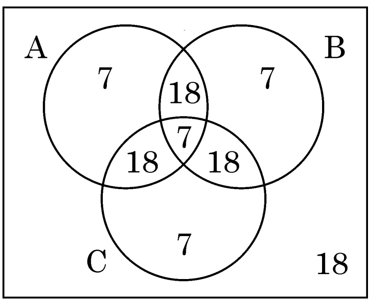
$$S_0 = \begin{bmatrix} 8.127 & 2.072 \\ 2.072 & 0.827 \end{bmatrix}, S_1 = \begin{bmatrix} 4.146 & 1.017 \\ 1.017 & 0.607 \end{bmatrix}$$

Given that $n_1 = 20$, $n_2 = 28$, test the null hypothesis $H_0 : \mu_1 = \mu_2$.

[Given that : $F_{2,\ 45,\ 0.01} = 5.11$].

(b) In some problems, it is desired to fit a plane having the equation $Z = \alpha + \beta X + \gamma Y$ to a set of data, consisting of the triplets (x_i, y_i, z_i) for $i = 1, 2, ..., n$. Fit a plane of this kind to the points (1, 1, 1), (0, 2, –5), (2, 1, 3), (3, 2, 2) and (4, 1, 12). 7

6\. (a) The following figure represents the sample space of an experiment which has 100 equally likely outcomes; the numbers in the figure, indicate the number of distinct outcomes contained in the respective sub-sets of the sample space. Show that the events A, B and C are pairwise independent but not mutually independent : 7



(b) The randon variable X has distribution function F given by :

$$F(x) = \begin{cases} 0, & x < 4 \\ 0.1, & 4 \le x < 5 \\ 0.4, & 5 \le x < 6 \\ 0.7, & 6 \le x < 8 \\ 0.9, & 8 \le x < 9 \\ 1, & x \ge 9 \end{cases}$$

Calculate the probabilities $P(X \le 6.5)$, $P(X > 8.1)$, $P(5 < x < 8)$. 8

7. (a) Let $\{X_n;\ n \geq 0\}$ be a branching process and if $E(X_1) = m$ (say), then prove that :

$$E(X_n) = m^n \text{ and } V(X_n) = \frac{m^{n-1}(m^n - 1)}{(m-1)}\sigma^2,$$

if $m \neq 1$ and equal to $n\sigma^2$ if ssss$m = 1$, where $\sigma^2 = V(X_1)$. 9

(b) Give the definition of a persistent and a transient state of a Markov chain. Also, define the first passage time distribution for the state k given that the system starts with j and mean recurrence time from state j to state k. 6

8. State whether the following statements are True *or* False. Justify your answer with short proof or a counter-example : 5×2=10

(i) The random variables X and Y have the joint p.d.f. given by :

$$f_{X,Y}(x, y) = 2;\ 0 < x, y < 1.$$

Then, X and Y are not independent random variables.

(ii) Ten chips numbered 1 through 10 are mixed in a bowl. Two chips numbered (X, Y) are drawn from the bowl, successively and without replacement. The probability that $X + Y = 10$ is 8/45.

(iii) Suppose that random variable X follows the following p.d.f. :

$$f(x) = \frac{1}{2\alpha}, -\alpha < x < \alpha, \alpha > 0.$$

The value of α so that the following are satisfied :

(a) $P[X > 1] = \frac{1}{3}$

(b) $P[X < \frac{1}{2}] = 0.7$

are respectively 3 and $\frac{5}{4}$.

(iv) Consider the following Markov chain with two states :

$$P = \begin{bmatrix} 0.2 & 0.8 \\ 0.7 & 0.3 \end{bmatrix}$$

with initial probability distribution $a^{(0)} = [0.5,\ 0.5]$. Then the second-step transition probability matrix, P^2 and the associated initial probability distribution $a^{(2)}$ are respectively :

$$P^2 = \begin{bmatrix} 0.60 & 0.40 \\ 0.45 & 0.55 \end{bmatrix},\ a^{(2)} = [0.375 \quad 0.625]$$

(v) The steady-state solution of the queuing model (M/M/1) : (N/FCFS) is :

$$P_n = \begin{cases} (1 - \lambda / \mu)\,(\lambda / \mu)^n, & \lambda / \mu \neq 1, 0 \leq n \leq N \\ 1 / (N+1) \qquad\qquad, & \lambda / \mu \neq 1 \end{cases}.$$

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