No. of Printed Pages : 8 **MMTE–005**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

December, 2025

MMTE-005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) There are six questions in this paper.*

(ii) The ***sixth*** *question is compulsory.*

(iii) Do any ***four*** *questions from Question Nos.* ***1*** *to* ***5****.*

(iv) Show all the relevant steps. Do the rough work at the bottom or at the side of the page only.

(v) Calculators are ***not*** *allowed.*

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1. (a) When do we say that the parity check matrix of a $[n, k]$ linear code is in standard form. Check whether the parity matrix :

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

of a linear code **C** is in standard form or not. What are the length and dimension of **C** ? 3

(b) Define the Hamming distance between two code words. What is the Hamming distance between the two code words 100110 and 110101 ? 2

(c) What is a repetition code ? If, in a repetition code in which the message of length three is sent thrice, the code word 110 110 100 is received, decode the message assuming there is at most one error. 2

(d) Let **C** be a binary cyclic code of length nine with generator polynomial $x^2 + x + 1$. Find the generator matrix and the parity check matrix of the code. 3

2. (a) Let $n \in \mathbf{N}$, q be a power of a prime and $0 \le s < n$. Define the q-cyclotomic coset of s modulo n. Find the 8-cyclotomic set of 1 modulo 17. 3

(b) Find the gcd of $x^5 + 2x^3 + 2x^2 + 3x + 1$, $x^4 + 2x^3 + x^2 + 3x + 1 \in \mathbf{F}_5[x]$. 4

(c) Let $\mathbf{C}_1$ be the [4, 3] binary linear code generated by :

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

and let $\mathbf{C}_2$ be the [4, 1]-binary linear code generated by [1010]. Let $\mathbf{C}$ be the code obtained through using $(u \mid u + v)$ construction on the codes $\mathbf{C}_1$ and $\mathbf{C}_2$. Find the generator matrix of $\mathbf{C}$. Also, give the length and the dimension of $\mathbf{C}$. 3

3. (a) Prove that the integers modulo n do not form a field if n is not a prime. 2

(b) The systematic generator matrix for a [6, 3] linear block code is $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$. Compute the standard array for syndrome decoding. 5

(c) Check whether the polynomial $x^3 + x + 1 \in \mathbf{F}_{32}$ is primitive. You are given that $x^{30} \equiv x^2 + 1 \pmod{x^3 + x + 1}$. You may assume that the polynomial is irreducible. 3

4. (a) Let **C** be the narrow-sense binary BCH code of designed distance δ = 5, which has a defining set T = {1, 2, 3, 4, 6, 8, 9, 12}. Let α be a primitive 15th root of unity, where $\alpha^4 = 1 + \alpha$ and let the generator polynomial of **C** be $g(x) = x^8 + x^7 + x^6 + x^4 + 1$. If $y(x) = x + x^4 + x^7 + x^8 + x^{11} + x^{13}$ received, find the transmitted code word. You can use Table 1. 6

Table 1

0000	0
0001	1
0010	α
0100	α^2
1000	α^3
0011	α^4
0110	α^5
1100	α^6
1011	α^7
0101	α^8
1010	α^9
0111	α^{10}
1110	α^{11}
1111	α^{12}
1101	α^{13}
1001	α^{14}

(b) Find the convolutional code (2, 1) with generator matrix G = [1, 1 + D], for the message $m = 1 + D + D^3$. 4

5. (a) Let **C** be the [7, 4, 2] binary code with the following parity check matrix : 5

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(i) Give the Tanner graph for this code.

(ii) List all the code words of **C** and hence find its minimum distance.

(b) List all the code words of the code **C** over $\mathbf{Z}_4$ generated by :

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Also, find the Lee weight distribution of the code. 5

6. Which of the following statements are true and which are false ? Justify your answer with short proof or a counter example.

5×2=10

(a) If F is a field and the polynomial $p(x) \in F[x]$ has no roots in F, then $p(x)$ is irreducible over F.

(b) The code with generator matrix :

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

has a unique code word of weight three.

(c) A quadratic code of length seven exists over $\mathbf{F}_3$.

(d) The parity check matrix of a turbo code can be the identity matrix.

(e) Every perfect code is self dual.

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