No. of Printed Pages : 7 **MPH–001**

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

December, 2025

MPH–001 : MATHEMATICAL METHODS IN PHYSICS

Time : 2 Hours *Maximum Marks : 50*

Note *: (i) Answer any* ***five*** *questions.*

(ii) The marks are indicated against each question.

(iii) Symbols have their usual meanings.

(iv) You may use a non-programmable calculator.

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1. Consider heat flow in a uniform bar of length L insulated along its length. The temperature of the bar is modelled by the one-dimensional heat diffusion equation :

4+6

$$\frac{\partial T(x, t)}{\partial t} = K \frac{\partial^2 T(x, t)}{\partial x^2} \quad (0 < x < L, t > 0)$$

(i) Separate the equation into two ODEs.

(ii) If the initial temperature distribution is given by :

$$T(x, 0) = (2L - x) \quad (0 < x < L)$$

solve the heat equation for the boundary conditions on $T(x, y)$ given as :

$$T(0,t)=0 \text{ and } \frac{\partial T(L,t)}{\partial x}=0,\ t\geq 0$$

2. (a) The generating function for Bessel's functions of the first kind and integral order is given by : 5

$$g(x,t)=\exp\left[\frac{x}{2}\left(t-\frac{1}{t}\right)\right]=\sum_{n=-\infty}^{+\infty} J_n(x)t^n$$

Show that :

$$J_0(x)+2J_2(x)+2J_4(x)+\ldots..$$

$$+2J_{2k}(x)+\ldots.=1$$

(b) The expression for Bessel's function of the first kind of order m is given by : 5

$$J_m(x)=\sum_{k=0}^{\infty}(-1)^k\frac{1}{k!\,\Gamma(m+k+1)}\left(\frac{x}{2}\right)^{2k+m}$$

Using this relation, derive the recurrence relation :

$$\frac{m}{x}\mathrm{J}_m(x)+\frac{d\mathrm{J}_m}{dx}=\mathrm{J}_{m-1}(x)$$

3. (a) Starting with generating function of Legendre polynomials of first kind : 4

$$g(x,t)=\left(1-2xt+t^2\right)^{-1/2}=\sum_{n=0}^{\infty}\mathrm{P}_n(x)t^n$$

show that :

$$\mathrm{P}_n(-x)=(-1)^n\,\mathrm{P}_n(x)$$

(b) Using the generating function of Hermite polynomials : 6

$$g(x,t)=e^{2xt-t^2}=\sum_{n=0}^{\infty}\frac{\mathrm{H}_n(x)t^n}{n!}$$

evaluate the integral :

$$\int_{-\infty}^{+\infty}xe^{-x^2}\mathrm{H}_n(x)\mathrm{H}_m(x)\,dx$$

4. (a) Show that the vectors $\left(\overline{V}_1, \overline{V}_2, \overline{V}_3\right)$,

$$\overline{V}_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \overline{V}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \overline{V}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

form the basis in R^3, the vector space of three-dimensional vectors. 4

(b) Let $A = \begin{bmatrix} 1 & -a & -b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -d & -e \\ 0 & 1 & -f \\ 0 & 0 & 1 \end{bmatrix}$

Calculate AB and show that it is also in form of upper triangular matrix. Calculate the value of d, e, f, so that AB = 1. Find the condition that AB = BA. 6

5. (a) Obtain the normalized eigen vectors for a real symmetric matrix : 6

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and find its diagonalizing matrix.

(b) Show that :

$$u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$$

is harmonic in some domain. Calculate its conjugate harmonic function $v(x, y)$ and construct the complex function $f(z)$. 4

6. (a) Determine the value of Gamma function $\Gamma\left(\frac{1}{2}\right)$. 6

(b) Determine and classify all the singularities of the following complex functions : 4

(i) $\dfrac{z}{e^{1/z} - 1}$

(ii) $\dfrac{1}{(2\sin z - 1)^2}$

7. Evaluate the integral $\int_{-\infty}^{+\infty} \frac{\cos x \, dx}{x^2 + a^2}$ by the method of Residues. 10

8. (a) Solve the initial value problem : 7

$$y'' + 5y' + 4y = f(t)$$

$$y(0) = 0, y'(0) = 0$$

with $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

using the methods of Laplace transforms.

(b) Define equivalence classes of a group. 3

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