

M. SC. (PHYSICS)
(MSCPH)
Term-End Examination
December, 2025

MPH-004 : QUANTUM MECHANICS-I

Time : 2 Hours

Maximum Marks : 50

Note : (i) *Attempt any five questions.*

(ii) *Symbols have their usual meanings.*

(iii) *The values of physical constants are given at the end.*

(iv) *You may use a calculator.*

1. (a) A thermal neutron with a speed v corresponding to the average thermal energy $\frac{3}{2}k_B T$ at $T = 300$ K is incident on a crystal. Will a diffraction pattern be observed ? Explain. 5

- (b) Use the uncertainty principle to derive an expression for the average size of the Hydrogen atom. 5
2. (a) In a region of space a particle of mass m and zero energy has a wave function $\psi(x) = Nxe^{-x^2/9}$ where N is a constant. Use the time-independent Schrödinger equation to derive the potential energy $V(x)$ of the particle. 5
- (b) Show that : 5

$$\left[\hat{x}^2, \hat{p}_x^2 \right] = 2i\hbar \left(\hat{x}\hat{p}_x + \hat{p}_x\hat{x} \right).$$

3. Consider a one-dimensional potential well with an infinite barrier at $x = 0$ and a finite potential V for $x \geq L$. (a) State the boundary conditions. (b) For a particle of mass m inside the well with an energy $E < V$, derive the quantization condition for the energy.

3+7

4. Write down the eigen value equations for the operators \hat{H} , \hat{L}^2 and \hat{L}_z for the hydrogen atom eigen function ψ_{nlm_l} . List the eigenfunctions for $n = 3$. What is the number of degenerate eigen functions for $n = 3$? For the ground state of the hydrogen atom which has the wave function :

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} \exp\left(\frac{-r}{a_0}\right)$$

calculate the expectation value of the potential energy $V(r) = \frac{-e^2}{r}$. You may use

$$\int_0^\infty e^{-ax} x^m dx = \frac{1}{a^{m+1}} m!. \quad 10$$

5. (a) Explain the importance of the simple harmonic oscillator potential in Physics. 2
- (b) Write down the Schrödinger equation for a one-dimensional simple harmonic oscillator of frequency ω and write the expression for its eigen energy E_n . 3

- (c) If the initial wave function for a simple harmonic oscillator is : 5

$$\psi(x, 0) = \frac{1}{\sqrt{3}}\psi_2(x) + i\sqrt{\frac{2}{3}}\psi_0(x)$$

determine $\psi(x, t)$.

6. (a) State any *two* properties of linear operators in Hilbert space. 2
- (b) Show that a linear combination of the degenerate eigen vectors of an operator belonging to a particular eigen value of the operator is also an eigen vector belonging to the same eigen value. 4
- (c) Define Hermitian and anti-Hermitian operator. For an operator \hat{A} , show that $(\hat{A} + \hat{A}^+)$ is Hermitian and $(\hat{A} - \hat{A}^+)$ is anti-Hermitian. 4
7. (a) $|\phi_1\rangle$ and $|\phi_2\rangle$ are the orthonormal basis states of a two-dimensional Hilbert space. The Hamiltonian \hat{H} in

this basis is given by the spectral representation :

$$\hat{H} = E_0 [|\phi_1\rangle \langle\phi_2| + |\phi_2\rangle \langle\phi_1|]$$

Determine the eigen values and normalized eigen vectors of \hat{H} . 5

- (b) Using the definition of the lowering operator $\hat{a} = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} + i\hat{p})$ derive the ground state wave function for the simple harmonic oscillator. 5

8. (a) Write the angular momentum eigen kets $(|j, m_j\rangle)$ for $j = \frac{1}{2}$. Derive the matrix elements for J^2 and J_z for $j = \frac{1}{2}$. 5

- (b) Describe briefly the Stern-Gerlach experiment and explain its principal result. 5

Physical constants :

$$h = 6.626 \times 10^{-34} \text{ Js}$$

$$\hbar = 1.015 \times 10^{-34} \text{ Js}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$m_n = 1.675 \times 10^{-27} \text{ kg}$$

$$k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

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