No. of Printed Pages : 8 **MPH–008**

MASTER OF SCIENCE (PHYSICS)

(MSCPH)

Term-End Examination

December, 2025

MPH-008 : QUANTUM MECHANICS—II

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any **five** questions.*

(ii) Symbols have their usual meanings.

1\. (a) Define the action of translation operator $\hat{T}(\varepsilon_x)$ on a position ket $|x\rangle$. Hence show that :

$$\langle x|\hat{T}(\varepsilon_x)|\Psi\rangle = \psi(x-\varepsilon_x)$$

where ε_x is an infinitesimal translation along x. 5

(b) What do you understand by a parity transformation ? Show that : 5

(i) the position operator $\hat{x}$ anti-commutes with the parity operator $\hat{\pi}$,

(ii) $(\hat{\pi})^2 = \hat{I}$.

2. The eigen functions for a particle in a one-dimensional box of size a are :

$$\Psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

with corresponding eigen energies :

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}.$$

(i) Write the eigen functions and energy eigen values for a system of two non-interacting particles confined to this one-dimensional box if the particles are distinguishable, identical bosons and identical fermions. 6

(ii) If the eigen energy of this system is $\frac{5\hbar^2\pi^2}{2ma^2}$, write the eigen function of the system if the two particles are identical fermions and if they are distinguishable particles. 4

3. (a) For a total angular momentum operator $\hat{J} = \hat{J}_1 + \hat{J}_2$, show that $\left[\hat{J}_x, \hat{J}_y\right] = i\hbar\,\hat{J}_z$. 4

(b) Write down the direct product kets :

$$\left|j_1, mj_1\right\rangle \otimes \left|j_2; mj_2\right\rangle$$

for the values of $j_1 = 1;\ j_2 = 1/2$, where the eigen values of $\hat{J}_1^{\,2}$ and $\hat{J}_2^{\,2}$ are $j_1(j_1+1)\hbar^2$ and $j_2(j_2+1)\hbar^2$ respectively. Also write down the eigen kets $\left|j, m_j\right\rangle$, where the eigen value of $\hat{J}^2$ is $j(j+1)\hbar^2$. 6

4. Calculate the first order correction to the ground state eigen function $\psi_1(x)$ and ground state energy eigen value E_1 for a particle of mass m in a one-dimensional infinite potential well of width L, perturbed by the potential : $H_1(x) = V\delta\left(x - \frac{L}{2}\right)$. Take

$$\psi_h(x) = \sqrt{\frac{2}{L}}\sinh\frac{\pi x}{L} \text{ and } E_h = \frac{\hbar^2\pi^2 n^2}{2mL^2}.$$ 4+6

5. (i) Consider a system with a Hamiltonian $\hat{H}$, for which the ground state eigen ket and energy eigen value are $|\phi_0\rangle$ and E_0 respectively. Show that for any trial ket $|\psi\rangle$: 6

$$E_0 \leq \frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle}$$

(ii) Can $\psi(x) = Nx\exp(-\alpha x^2)$, where α and N are constants, be used as a trial wave function to determine the upper bound to the energy of the first excited state of a simple harmonic oscillator potential ? Explain. 4

6. Consider a two-state system described by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}(t)$, where $|1\rangle$ and $|2\rangle$ are the two eigen kets of $\hat{H}_0$ with :

$$\hat{H}_0 |1\rangle = E_1 |1\rangle ; \quad \hat{H}_0 |2\rangle = E_2 |2\rangle$$

and

$$V(t) = V_0 \left[e^{i\omega t} |1\rangle \left\langle 2 \right| + e^{-i\omega t} |2\rangle \langle 1| \right]$$

(i) Obtain the matrix elements : 2

$$V_{ij} = \left\langle i \middle| V_{(t)} \middle| j \right\rangle \text{ with } i, j = 1, 2.$$

(ii) The wave function of the system at time t is : 2

$$\psi(t) = a_1(t)\, e^{\frac{-iE_1 t}{\hbar}} \left|1\right\rangle + a_2(t)\, e^{\frac{-iE_2 t}{\hbar}} \left|2\right\rangle$$

If :

$$i\hbar \frac{\partial a_1}{\partial t} = a_1 V_{11} + e^{i\omega_{12} t}\, V_{12}\, a_2$$

$$i\hbar \frac{\partial a_2}{\partial t} = e^{-i\omega_{12} t}\, V_{21}\, a_1 + V_{22}\, a_2$$

where :

$$\omega_{12} = \frac{E_1 - E_2}{\hbar}$$

derive the differential equations for a_1, a_2 for the given potential.

(iii) Using the initial conditions :

$$a_1(t=0)=1\,;\quad a_2(t=0)=0$$

solve the differential equations obtained in (ii) to calculate the transition probability $|a_2(t)|^2$ at time t. 6

7. (a) Write the expression for the probability of a system to make a transition from a state $|i\rangle$ at $t = 0$ to a state $|n\rangle$ at a later time t under the action of a perturbation V(t), upto first order in perturbation theory. Use this to calculate the transition probability for a static potential switched on at $t = 0$: 5

$$V = \begin{cases} 0, & t<0 \\ V, & t>0 \end{cases}$$

(b) Calculate the low energy differential and total scattering cross-section for the following scattering potential : 5

$$V(r) = \begin{cases} V_0, & r \le R_0 \\ 0, & r > R_0 \end{cases}$$

8. Write Dirac's equation for a free particle. Derive the expressions for the probability density and probability current density. 10

× × × × ×