No. of Printed Pages : 6 **MST–012**

# M. SC. (APPLIED STATISTICS)

# (MSCAST)

## Term-End Examination

## December, 2025

## MST-012 : PROBABILITY AND PROBABILITY DISTRIBUTIONS

*Time : 3 Hours* *Maximum Marks : 50*

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***Note :*** *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions from the remaining question numbers **2** to 6.*

*(iii) Use of scientific calculator (non-programmable) is allowed.*

*(iv) Symbols have their usual meanings.*

1. (a) Let $\Omega$ be the sample space of the random experiment of tossing a coin twice. Let X denotes the number of heads in two tosses and Y denotes the number of tails before the first head. Prepare the joint probability distribution of X and Y. 2

(b) Suppose in a party there are $n$ (< 365) persons. Assume that each of them is born in a non-leap year. Also, assume that birth rate is uniform throughout the 365 days of a year. Find the probability of at least one sharing birthday. 2

(c) Obtain MGF of continuous uniform distribution. 3

(d) If X~Expo(2), then find upper bound of $P(X \geq 2)$. 3

2. Define induced probability space by a random variable on the real line. 10

3. (a) Joint PDF of the jointly continuous random variable (X, Y) is given by :

$$f_{X,Y}(x, y) = \begin{cases} \lambda_1, \lambda_2 \, e^{-(\lambda_1 x + \lambda_2 y)}; & x, y > 0 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

Find the PDF of the random variable $\frac{X+Y}{2}$. 8

(b) If PMF of a random variable X is given by :

| X | $p(x)$ |
|---|---|
| 0 | 0.1 |
| 1 | 0.4 |
| 2 | 0.3 |
| 3 | 0.2 |

find the PMF of the random variable $2X + 7$. 2

4. (a) If $X \sim \text{Pois}(2)$ and $Y \sim \text{Pois}(3)$ are two independent random variables, then find $P(X + Y < 4)$. 4

(b) If $X \sim \text{Beta}(4, 5)$, then write the PDF and CDF of X. 3

(c) If $X \sim \text{Gamma}(100, 5)$, then write the PDF and CDF of X. 3

5. (a) If $X \sim \text{Lap}(30, 5)$, then find probability that the random variable X is less than 28. 5

(b) If $X \sim LN(\mu, \sigma^2)$, then prove that : 5

$$F_X(\mu) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right)$$

6. Before the result of the election between two candidates A and B, we do not know what percent of votes candidates A and B will get. Suppose candidate A gets $p$ percent of votes and candidate B gets $q$ percent of votes, where $p + q = 100$ percent. Values of $p$ and $q$ are unknown before the results of the election. Suppose you being a statistician are given the job to estimate the value of $p$ prior to election. Suppose your organisation wants that your estimate of $p$ can be at the most 3 percent away from the true value of $p$. To meet the requirement of your organisation you need to conduct a random survey of the voters where you have to record the responses as 'yes' or 'no' in favour

of candidate A (say). Assume that every person of your sample gives his/her responses as ‘yes’ or ‘no’ only. How large size of the sample you will need to make your estimate within 3 percent of the true value of $p$ with a probability of at least 0.95 ? 10

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