No. of Printed Pages : 4 **MST–022**

M. SC. (APPLIED STATISTICS) (MSCAST)

Term-End Examination

December, 2025

MST-022 : LINEAR ALGEBRA AND MULTIVARIATE CALCULUS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No.* **1** *is compulsory.*

(ii) Attempt any **four** *questions from the remaining Question Nos.* **2** to **6**.

(iii) Use of scientific (non-programmable) calculator is allowed.

(iv) Symbols have their usual meanings.

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1. State whether the following statements are True *or* False. Give reasons in support of your answers : 5×2=10

(a) The equation of the level surface of the function $f(x, y, z) = \sqrt{y^2 - xz} + 3y$ through the point (1, 4, 7) is $\sqrt{y^2 - xz} + 3y = -3$.

(b) The function :

$$W = \{(x, y, 2x + 3y) : x,\ y \in \mathbf{R}\}$$

is a subspace of $\mathbf{R}^3(\mathbf{R})$.

(c) The value of $h \in \mathbf{R}$ for which the set of vectors {(2, h, 4), (5, h, 2), (3, h, 0) is linearly dependent is 0.

(d) The directional derivative of the function $f : \mathbf{R}^3 \to \mathbf{R}$ is defined by $f(x, y, z) = 2xy^2 + yz + z^2$ at Q(1, 1, 0) in the direction of $\vec{a} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}k$ equals to – 1.

(e) The matrix :

$$A = \begin{Bmatrix} 1 & -1 \\ 0 & 1 \end{Bmatrix}$$

satisfies $A^4 = I_{2\times2}$.

2. Find SVD of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$. 10

3. Find an orthonormal basis for the subspace of $\mathbf{R}^4$ having basis : 10

$$w_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, w_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, w_3 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 4 \end{bmatrix}.$$

4. (a) If $\ln z = z^2 + 2y - x$, then find the value of $\frac{\partial z}{\partial y}$ at (1, 0, 2). 5

(b) Find $\frac{\partial w}{\partial r}$ when $r = 1$, $s = -1$, if $w = (x + y + z)^2$, $x = r - s$, $y = \cos(r + s)$, $z = \sin(r + s)$. 5

5. (a) If :

$$U(x, y) = x^2 y \tan^{-1}\left(\frac{y}{x}\right) + xy^2 \sin^{-1}\left(\frac{x}{y}\right); x, y > 0,$$

then find the value of : 5

$$x^2 U_{xx} - xU_x + 2xyU_{xy} - yU_y + y^2 U_{yy}.$$

(b) Using steepest descent algorithm, find the value of the function :

$$f(x) = 2x^2 + 2xy + y^2 + x - y$$

at the second iteration, starting with initial guess as $X_0 = (0, 0)^T$. 5

6. Explain how principle of gradient descent works. 10

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