No. of Printed Pages : 6 **MSTE-012**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

December, 2025

MSTE-012 : STOCHASTIC PROCESSES

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions from the remaining question nos. 2 to 6.*

(iii) Use of scientific (non-programmable) calculator is allowed.

(iv) Symbols have their usual meanings.

---

1. State whether the following statements are True *or* False. Give reasons in support of your answers : 5×2=10

   (a) If parametric space 'T' is continuous and state space 'S' is discrete, then the stochastic process is a Discrete valued stochastic process.

   (b) The sum of two independent Poisson processes $X_1(t)$ and $X_2(t)$ with parameters $\lambda_1(t)$ and $\lambda_2(t)$, respectively is also a Poisson process with parameter $(\lambda_1 + \lambda_2)t$.

   (c) The average waiting time of a customer in the queue $E(Wq)$ is $\left(\frac{\lambda}{4}\right)$ times of the average length of the queue Lq.

   (d) The complex random process $Z(t) = X(t) + iY(t)$ is said to be Wide

Sense Stationary if {X(t)} and {Y(t)} are jointly stationary in strict sense.

(e) If the one-step transition probability does not depend on the step, i.e., $p_{iJ}(n-1, n) = p_{iJ}(m-1, m)$, the Markov chain is called a heterogeneous Markov chain.

2. (a) What is meant by Transition Probability in stochastic processes ? Considering a discrete-valued stochastic process, define (i) one-step, (ii) three-step and (iii) r-step transition probabilities. 5

(b) Find the probability that a random walk of 12 steps with probability $p = 0.8$ ends at (i) position $x = 6$, (ii) position $x = -4$. 5

3. (a) Explain what is meant by a pure birth process. Obtain the probability distribution of pure birth process. 5

(b) In the standard gambler's ruin problem, with total stake 'a' and gambler's stake 'k' and gambler's probability of winning at each play is p, calculate the probability of ruin in the following case :

$a = 80$, $k = 70$ and $p = 0.45$.

Also find the expected duration in each case. 5

4. (a) Why there are two types of renewal processes ? What are the basic differences between them ? 5

(b) Find the mean and variance of the stationary process $\{X(t)\}$, whose Auto Correlation Function is given by : 5

$$R_{XX}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$$

5. (a) What do the letters in the symbolic representation $(a/b/c)$: (d/e) of a queueing model represent ? Also write down the formulas for P_0 and P_n in a Poisson queue system in the steady state. 4

(b) Customer arrive at a one-man barber shop according to a Poisson process with a mean inter-arrival time 12 minutes. Customers spend an average of 10 minutes in the barber's chair. Then : 6

(i) What is the expected number of customers in the barber shop and in the queue ?

(ii) Calculate the percentage of time, an arrival can walk straight into the barber's chair without waiting.

(iii) How much time can a customer expect to spend in the barber's shop ?

(iv) What is the average time customers spend in queue ?

(v) What is the probability that the waiting time in the system is greater than 30 minutes ?

6. (a) Derive the mean and variance of the Pure-Birth Process. 5

(b) The Transition Probability Matrix of a Markov chain $\{X_n\}$, $n = 1, 2, 3, ...$, having 3 states 1, 2 and 3 is : 5

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

and the initial distribution is

$$P^o = (0.7, 0.2, 0.1).$$

Find (i) $P\{X_2 = 3\}$ and

(ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

× × × × ×