

Ph. D. IN STATISTICS
(PHDSTAT)
Term-End Examination
December, 2025

RST-005 : STATISTICAL INFERENCE IN LIFE
TESTING

Time : 3 Hours

Maximum Marks : 100

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt one **four** questions from question Nos. 2 to 7.*

(iii) *Scientific calculator (non-programmable) is allowed.*

(iv) *Symbols have their usual meanings.*

1. (a) State whether the following statements are True or False. Give reasons in support of your answers : $5 \times 2 = 10$

- (i) Bayes estimator under the SELF in median of the posterior distribution.
- (ii) Gibbs sampling generates random numbers using the joint distribution.
- (iii) If a researcher terminates the experiment after a fix number of failures then the censoring scheme is called time censoring.
- (iv) If we have a random number 13 and the next random number using middle square method will be 16.
- (v) A system has *two* independent components which are connected in series configuration. If the reliabilities of these components are 0.5 and 0.3 respectively, then the reliability of the system will be 0.08.

- (b) Generate the successive random numbers using linear congruential generator :

$$X_i = (aX_{i-1} + C) \bmod m$$

with parameter $a = 5, C = 3, m = 16$ and starting value $X_0 = 7$. Stop when the repetitions starts. 10

2. (a) If a researcher observed that the reduction of weight (in kg) after a diet plan follows exponential distribution with parameter θ whose pdf is given as follows : 10

$$f(x) = \theta e^{-\theta x}; x > 0, \theta > 0$$

then

- (i) Find the Cramer-Rao lower bound for the variance of the estimator.
- (ii) Also find UMVUE of $\frac{1}{\theta}$

- (b) Describe Lehman-Scheffe's theorem. 10

3. (a) The number of the customer arrives at a restaurant follows Poisson distribution whose pmf is given as follows :

$$P[X = x] = \frac{e^{-\lambda} \lambda^x}{x!}; x = 0, 1, 2, \dots, \lambda > 0$$

It is assumed that the arrival rate (λ) of the customers is a random variable and has gamma prior whose pdf is given as follows :

$$f(\lambda) = \frac{b^a}{\Gamma a} e^{-b\lambda} \lambda^{a-1}; a, b, > 0$$

If X_1, X_2, \dots, X_n represent the number of customer arrives at the restaurant, then show that gamma is a conjugate prior.

10

- (b) Describe 4 loss functions in Bayesian analysis. 10

4. Describe metropolis-Hasting's algorithm and write R-program to simulate 10000 random numbers from $N(\mu, 1)$ using $U(0, 1)$ as proposal density. 20

5. (a) random number generator produces the following $U(0,1)$ random numbers : 10

0.62 0.72 0.57 0.90 0.12 0.72

0.19 0.52 0.10 0.82 0.61 0.52

0.32 0.62 0.27 0.73 0.32 0.50

0.72 0.91 0.21 0.72 0.41 0.34

0.45

Apply the Chi-square goodness of fit test to that random numbers follow $U(0,1)$ distribution at 5% level of significance.

(Given $\chi_{(4)}^2, 0.05 = 9.49, \chi_{(5)}^2, 0.05 = 11.07$).

- (b) Describe Inversion method of generating random numbers from continuous distribution. Also write its algorithm. 10
6. (a) If failure density function of a unit is given as follows :

$$f(t) = \begin{cases} 0.012e^{-0.012t}; & t \geq 0 \\ 0 & ; \text{ otherwise} \end{cases}$$

then calculate

- (i) reliability function
- (ii) mean time to failure
- (iii) medium failure time
- (iv) life of the unit, if the reliability of 0.96 is desired. 2+2+3+3

- (b) Describe Reliability function and Hazard rate. 10

7. Write short notes on any *two* of the followings : 10+10

- (i) Properties of good estimators
- (ii) Progressive censoring
- (iii) Classical and Bayesian approaches

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