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M. SC. (MATHEMATICS WITH APPLICATIONS TO COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

June, 2025

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours *Maximum Marks :25*

Note : *(i) Question No. **5** is compulsory.*

*(ii) Do any **three** questions from Q. Nos. **1** Q. **4**.*

(iii) Use of calculator is not allowed.

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1. (a) Let T : $\mathbf{R}^3 \rightarrow \mathbf{R}^2$ be a linear transformation defined by :

$$T(x, y, z) = (x + y - z, x + y + 3z)$$

Find the matrix of T relative to the ordered bases {(1, 1, 1), (1, 1, 0), (1, 0, 1)} of $\mathbf{R}^3$ and {(1, 1), (0, 1} of $\mathbf{R}^2$. 2

(b) Find the quadratic polynomial that best fits the data (– 2, –4), (–1, –4), $\left(1, \frac{11}{5}\right)$ and (2, 8). 3

2. Let :

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

Find a matrix P such that $P^{-1}AP$ is in block diagonal form. 5

3. Find the singular value decomposition of A, where : 5

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$

4. (a) Prove that if N is a non-zero nilpotent operator, then N is not diagonalisable. 2

(b) Check whether or not $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$ is unitarity diagonalisable. If it is, find a unitary matrix U such that U* AU is diagonal. If A is not unitarily diagonalisable, obtain the Schur decomposition of A. 3

5. Which of the following statements are true and which are false ? Justify your answers with a short proof or a counter-example, whichever is appropriate : 10

(i) If two matrices have the same characteristic polynomial, they are similar.

(ii) An invertible matrix must be positive definite.

(iii) A and AA^t have the same rank for any matrix A.

(iv) For every matrix $S \in M_n(\mathbf{R})$, there is an $n \times n$ orthogonal matrix O such that $O^t SO \in M_n(\mathbf{R})$ is upper triangular.

(v) Any non-zero square matrix over **R** has positive spectral radius.

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