No. of Printed Pages : 6 **MMT–004**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2025

MMT-004 : REAL ANALYSIS

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions out of question nos. **2** to **6**.*

(iii) Use of calculator is not allowed.

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1. State whether the following statements are True *or* False. Give reason for your answers : 5×2=10

(a) If A is a subset of a metric space (X, d) and $x \notin A$, then $d(x, A) \neq 0$.

(b) The sequence $\left\{\left(\frac{1}{n}, \frac{1}{n}\right) n = 1, 2, ...\right\}$ is not convergent in $\mathbf{R}^2$ under the discrete metric on $\mathbf{R}^2$.

(c) $\mathbf{R}^2 \setminus \{(0, 1)\}$ is path connected.

(d) The partial derivative of the function $f(x, y) = (x^2 - y, x + 2y)$ at (1, 1) is (2, 1).

(e) Every Borel set is measurable.

2. (a) Let A be a subset of a metric space (X, d). Prove that : 2

$$\overline{A} = \{x \in A : d(x, A) = 0\}$$

(b) Prove that a Cauchy sequence in a metric space is convergent if and only if it has a convergent subsequence.

Is the continuous image of a Cauchy sequence a Cauchy sequence. Justify your answer. 5

(c) For the function $F : \mathbf{R}^2 \to \mathbf{R}^3$ given by $F(x_1, x_2) = (e^{x_1}, e^{x_2}.x_1 + x_2)$, find the directional derivative in the direction of (1, 1) at (0, 0). 3

3. (a) Prove that a finite union of compact sets in a metric space is compact. What about arbitrary union ? Justify your answer. 5

(b) Define the outer measure in * and measure m of a subset A of **R**. Prove that for any given $\varepsilon > 0$, we can find an open set O such that $A \subset O$ and $m(O) \leq m^*(A) + \varepsilon$. 5

4. (a) Apply the implicit function theorem to the function $f(x, y, z) = x^2 + y^3 - xy \sin z$ at the point (1, –1, 0). 3

(b) Prove that a metric X is connected if every continuous function f on X to the discrete metric space {1, –1} reduces to a constant function. 3

(c) Find the critical points of the function :

$$f(x, y, z) = x^2 y + y^2 z + z^2 - zx$$

and check whether they are extreme points. 4

5. (a) Prove that a path connected metric space is connected. What about the converse ? Justify your answer. 4

(b) State and prove the dominated convergence theorem. 6

6. (a) For a real function $f \in L_1[-\pi, \pi]$, define the *n*th Fourier coefficient of *f*, the exponential and the trigonometric forms of the Fourier series of *f*.

Find the Fourier series of the function : 4

$$g(t) = \begin{cases} -1, & -\pi < t < 0 \\ 1, & 0 \le t < \pi \end{cases}$$

(b) Suppose that *f* and *g* are two integrable functions such that $\int_A f\, dm \le \int_A g\, dm$ for all $A \in \boldsymbol{m}$. Show that $f \le g$ a.e. 3

(c) Find and classify the extreme values of the function : 3

$$f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

subject to the constraint :

$$x_1 + 2x_2 = 2, \; x_1, x_2 \geq 0.$$

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