No. of Printed Pages : 5 **MMT–005**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2025

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **three** questions from Question Nos. **2** to **5**.*

(iii) Use of calculator is not allowed.

1. State, giving reasons whether the following statements are true *or* false : 5×2=10

(a) The principal argument of the complex number $-1 - i$ is $\frac{5\pi}{4}$.

(b) The function $f(z) = z - \bar{z}$ is differentiable at $z = 0$.

(c) The identity :

$$\log(z_1 \, . \, z_2) = \log z_1 + \log z_2$$

holds for every pair of complex numbers z_1 and z_2.

(d) $\int_{C_1} \frac{1}{z^2+4} dz = \int_{C_2} \frac{1}{z^2+4} dz$, where C_1 and C_2 denote positive oriented circles $|z| = 3$ and $|z| = 4$ respectively.

(e) The function $f(z) = z^2 + 2z$ is conformal everywhere.

2. (a) Prove that the function : 3

$$u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$

is harmonic. Find its harmonic conjugate.

(b) Find the singular points and their nature for the function : 2

$$f(z) = \frac{1}{z^6}\left(z - \frac{z^3}{3!} - \sin z\right)$$

3. (a) Find the real and imaginary parts of the function : 2

$$f(z) = e^{z^2}$$

(b) (i) Find the fixed points of the bilinear transformation : $1\frac{1}{2}$

$$w(z) = \frac{-2 + (2+i)z}{i + z}$$

(ii) Prove that, if the sequence (z_n) converges to z, then the sequence $(\bar{z}_n)$ converges to $\bar{z}$. $1\frac{1}{2}$

4. (a) Find the Laurent's series expansion in powers of z for the function : 3

$$f(z) = \frac{1}{z^2 - z - 2}$$

in the region $1 < |z| < 2$. Also write its principal part at $z = 0$.

(b) Without evaluating the integral, show that : 2

$$\left| \int_C \frac{dz}{z^2 + 1} \right| \leq \frac{3\pi}{16}$$

where C is the arc of circle $|z| = 3$ from $z = 3$ to $z = 3i$.

5. (a) Use Cauchy Residue theorem to evaluate : 3

$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}, \quad a^2 < 1$$

(b) Find the radius of convergence of the power series : 2

$$\sum_{n=1}^{\infty} \left[3 + (-1)^n\right]^n z^n$$

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