No. of Printed Pages : 7 **MMT–007**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2025

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Weightage : 50%

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions out of Q. Nos. **2** to **7**.*

(iii) Use of scientific/non-programmable calculator is allowed.

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1. State whether the following statements are True *or* False. Justify your answer with the help of a short proof or a counter-example :

5×2=10

(a) For initial value problem :

$$y' = \begin{cases} \dfrac{2y}{x}, & x > 0 \\ 0, & x = 0 \end{cases}, \qquad y\,(0) = 0$$

the Lipschitz condition is not satisfied in any closed rectangle containing (0, 0).

(b) $L\left\{e^{2at} \cos 3\omega t\right\} = \dfrac{s}{s^2 + 9\omega^2}$, where L is the Laplace transformation.

(c) The initial value problem $y' = \dfrac{y - x}{y + x}$, $y\,(1) = 2$ has a solution 2.0596 at $x = 1.2$, using Runge-Kutta second order method with $h = 0.1$.

(d) $f(x) = \begin{cases} e^{-ax}; & x \geq 0, a > 0 \\ 0; & x < 0 \end{cases}$

The Fourier transform of the function $f(x)$ is a complex function.

(e) Finite element Galerkin method is a weighted residual method and does not require the variational form of the given differential equation.

2. (a) Prove that : 5

$$\int_{-1}^{1} \frac{P_n(x)}{\sqrt{1-2xt+t^2}}\, dx = \frac{2t^n}{2n+1}$$

(b) Prove the following relation for Hermite polynomials $H_n(x)$: 5

$$xH'_n(x) = nH'_{n-1}(x) + nH_n(x)$$

3. (a) Obtain a series solution about $x = 0$ of the equation : 5

$$x^2 y'' + 6xy' + (6 + x^2)y = 0$$

(b) Using Green's function, solve the boundary-value problem : 5

$$\left(\frac{d^2 y}{dx^2} + y\right) = -x, \quad y(0) = y\left(\frac{\pi}{2}\right) = 0.$$

4. (a) Using Laplace transform, solve the PDE : 6

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0$$

subject to the conditions :

$$u\,(0,\,t) = 10 \sin 2t$$

$$u\,(x,\,0) = 0$$

$$u_x(x, 0) = 0,$$

$$\lim_{x \to \infty} u(x, t) = 0.$$

(b) Solve the initial value problem $y' = x^2 + y^2$, y (0) = 1 upto x = 0.4 using fourth order Taylor's series method with $h = 0.2$. 4

5. (a) Show that the Milne-Simpson method : 6

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left(y'_{n+1} + 4y'_n + y'_{n-1}\right)$$

is not absolutely stable for any h and for any initial value problem.

(b) Using the generating function for $J_n(x)$, prove that : 4

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$$

for integer values of n. Here J_n is the Bessel's function of the first kind of order n.

6. (a) Find the solution of the following initial boundary value problem, subject to the given initial and boundary conditions : 6

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1,$$

$$u\ (x,\ 0) = \sin\ (2\pi x)$$

$$u(0, t) = 0 = u(1, t)$$

Assume step length in x-direction h = 0.25, solve using Laasonen method with mesh ratio λ = 0.6. Integrate for two time levels.

(b) Find the inverse Laplace transform of :4

$$\frac{21s - 33}{(s+1)(s-2)^3}$$

7. (a) The heat conduction equation $u_t = u_{xx}$ is approximated by : 5

$$\frac{1}{2k}\left(u_m^{n+1} - u_m^{n-1}\right) = \frac{1}{h^2}\left(u_{m-1}^n - 2u_m^n + u_{m+1}^n\right)$$

Investigate the stability using the Von Neumann method.

(b) Solve the boundary value problem :

$$y'' - 4y' + 3y = 0; \quad y(0) = 1,\ y(1) = 0$$

using second order finite difference method with $h = \frac{1}{3}$. 5

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