No. of Printed Pages : 12 **MMT–008**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2025

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 Hours *Maximum Marks : 100*

Weightage : 50%

Note : *(i) Question No. 8 is compulsory. Attempt any* ***six*** *questions from question nos.* ***1*** *to* ***7****.*

(ii) Use of scientific and non-programmable calculator is allowed.

(iii) Symbols have their usual meanings.

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1. (a) Students in a certain college subscribe to three news magazines A, B and C according to the following proportions :

A : 20%, B : 15%, C : 10%

both A and B : 5%, both A and C : 4%, both B and C : 3%, all three A, B and C : 2%. If a student is chosen at random, what is the probability he/she subscribes to none of the news magazines ? 6

(b) The lifetime of an automobile battery is described by a random variable X having the negative exponential distribution, $f(x) = \lambda e^{-\lambda x}$, $x > 0$ and $\lambda = \frac{1}{3}$. Then : 9

(i) Calculate the probability that the lifetime of the battery will be between 2 and 4 units.

(ii) If the battery has lasted for 3 time units, what is the conditional probability that it will last for at least an additional time unit ?

2. (a) The random variables X and Y have the joint probability density function (p.d.f.), $f_{X,\,Y}$ given by : 8

$$f_{X,\,Y}(x,\,y)=\frac{6}{7}\left(x^2+\frac{xy}{2}\right),$$

$$0<x<1,\,0<y<2$$

Find :

(i) The marginal p.d.f.'s f_X, f_Y and the conditional p.d.f. $f_{Y/X}(\cdot/x)$.

(ii) E(Y) and $E(Y\,|\,X=x)$

(iii) The probability $P\left[Y>\frac{1}{2}\,|\,X<\frac{1}{2}\right]$.

(b) Give the joint p.d.f. of the random variables X and Y :

$$f_{X,\,Y}(x,y)=2;\quad 0<x<y<1,$$

determine whether the random variables are independent or not. 7

3. (a) Let a large number of containers of candy are made up of two types of candy, say A and B. Type A contains 70% sweet and 30% sour ones while for type B, these percentages are reversed. Furthermore, suppose that 60% of all candy jars are of Type A while the remainder are of Type B. A jar of unknown type was given to you and you are allowed to sample one piece of candy. Let the picked candy was sweet. Find the probabilities that : 7

(i) it is from Type A; and

(ii) it is from Type B.

(b) In one stage of the development of a new medication for an allergy, an experiment is conducted to study how different doses of the medication affect the duration of relief from the allergic symptoms. 10 patients are included in the experiment. Each patient receives a specific dose of the medication and is asked to report back as soon as the protection of the medication seems to be wear off.

The observations are recorded and are presented below showing the dose (X) and respective duration of relief (Y) :

X	**Y**
3	9
3	5
4	12
5	9
6	14
6	16
7	22
8	18
8	24
9	22

Find the least square estimates, $\hat{\beta}_1$ and $\hat{\beta}_2$ of the parameters of the fitted regression line $y = \beta_1 + \beta_2 x$ and draw the scatter diagram of the data along with the fitted regression line. 8

4. (a) Obtain the waiting time distribution for the (M/M/1) : (∞/FIFO) queuing system.

If customers arrive at a rate of 6 per hour in a company for getting some services and the clerk sitting on the counter can service 10 customers on the average per hour, what will be the average waiting time of a customer in the system before getting service ? 6

(b) In an experiment, two independent observers, 1 and 2, collected data on two different samples of sizes 10 each. They had the mean vectors and covariance matrices as follows :

$\bar{x}_1' = [0.3781, 0.3755]$

$\bar{x}_2' = [0.3772, 0.3750]$

$$S_1 = \begin{bmatrix} 2.64 & 3.18 \\ 3.18 & 11.42 \end{bmatrix};$$

$$S_2 = \begin{bmatrix} 6.48 & 0.45 \\ 0.45 & 8.26 \end{bmatrix}$$

Assuming that the data follow bivariate normal distribution, test the hypothesis at 95% level of significance that the population mean vectors of the observers are the same. [Given that : $F_{2, 17, 0.05} = 3.59$] 9

5. (a) Let us consider a source of radioactive material which is emitting α-particles. Let X_t be defined as the number of particles emitted during a specified time period [0, t]. Stating the postulates of the Poisson process, find the probability, $p_n(t) = P[X_t = n]$, $n = 0, 1, 2, ...$ Hence or otherwise, if X and Y are independent Poisson variates, show that the conditional distribution of X given X + Y is binomial. 7

(b) Define a branching process and explain it with the help of at least *one* example. Let ξ_r be the random variable denoting the number of individuals produced in the r^{th} generation with the distribution :

$$P[\xi_r = k] = p_k, \ k = 0, 1, 2...; \ \Sigma p_k = 1$$

and let X_n be the population size of the nth generations with $X_0 = 1$, then show that the generating function of X_{n+1} satisfies the relation : 8

$$G_{n+1}(s) = G_n[G(s)]$$

6. (a) Let $\{A_j, j = 1, ..., 5\}$ be a partition of the sample space S and suppose that :

$$P(A_j) = \frac{j}{15}$$

and

$$P(A \mid A_j) = \frac{5-j}{15}, j = 1, 2, ..., 5.$$

Compute the probabilities $P(A_j \mid A)$, $j = 1, 2, ..., 5.$ 7

(b) Define the renewal function and renewal density.

Let $\{X_n, n = 1, 2, ...\}$ be a sequence of non-negative independent variables and

$$S_n = X_1 + X_2 + ... + X_n, \quad n \geq 1$$

be the time upto the nth event with $S_0 = 0$. Further, let $F_n(x) = P[S_n \leq x]$ be the distribution function of $S_n, n \geq 1$. Then show that the distribution of N(t), the number of events in the interval (0, t], is given by : 8

$$p_n(t) = P[N(t) = x] = F_n(t) - F_{n+1}(t)$$

7. (a) Consider the following Markov chain with two states :

$$P = \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix}$$

with initial distribution $\alpha^{(0)} = [0.7\ 0.3]$. Then determine the second-step, fourth-step and eighth-step transition probability matrices, P^2, P^4 and P^8 along with initial distributions $\alpha^{(2)}, \alpha^{(4)}$ and $\alpha^{(8)}$. 7

(b) Let X and Y be random variables with joint p.d.f. given by : 8

$$f_{X, Y}(x, y) = \frac{6}{5}(x^2 + y), \ 0 \le x \le 1, 0 \le y \le 1$$

(i) Determine the marginal p.d.f.'s f_X and f_Y.

(ii) Investigate whether or not the random variables X and Y are independent. Justify your answer.

8. State whether the following statements are TRUE or FALSE. Justify your answer with a short proof or a counter-example : 2×5=10

(i) Let the joint p.d.f. of random variables X and Y be

$$f(x, y) = e^{-(x+y)}, \quad x \ge 0, y \ge 0.$$

Then X and Y are independent variables.

(ii) Given the following joint probability function of X and Y and the respective marginal distributions, $p_1(x)$ and $p_2(y)$. The Cov (X, Y) is obtained as 10 :

y \ x	1	2	3	4	5	$p_2(y)$
1	0.10	0.0	0.0	0.0	0.0	0.1
2	0.0	0.2	0.0	0.1	0.0	0.3
3	0.0	0.0	0.2	0.0	0.0	0.2
4	0.0	0.1	0.0	0.2	0.0	0.3
5	0.0	0.0	0.0	0.0	0.1	0.1
$p_1(x)$	0.1	0.3	0.2	0.3	0.1	

(iii) For the first-step transition probability matrix (t.p.m.) $P = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix}$, the fourth-step t.p.m. will be

$$P^4 = \begin{bmatrix} 0.4281 & 0.5719 \\ 0.4274 & 0.5726 \end{bmatrix}$$

(iv) Let a gambler with initial capital k plays against an opponent with initial capital $a-k$. Then the probability of gambler's rain is given by $\frac{(q/p)^{a'}(p/q)^k}{(q/p)^a+1}$ if $p \neq q$.

(v) If the random variables X_1 and X_2 have the bivariate normal distribution with parameters $\mu_1 = -1$, $\mu_2 = 3$, $\sigma_1^2 = 4$, $\sigma_2^2 = 9$ and $\rho = 1/2$, then with two constants C_1 and C_2, $E(C_1X_1 + C_2X_2) = -C_1 + 3C_2$ and $V(C_1X_1 + C_2X_2) = 4C_1 + 9C_2 + 6C_1C_2$.

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