No. of Printed Pages : 6 **MMTE–001**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M.SC. (MACS)]

Term-End Examination

June, 2025

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Answer any* ***four*** *questions from Q. Nos.* ***2*** *to* ***7****. Symbols have their usual meanings.*

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1. State whether the following statements are true *or* false. Justify your answers with a short proof or a counter-example : 5×2=10

   (i) Every tree is a bipartite graph.

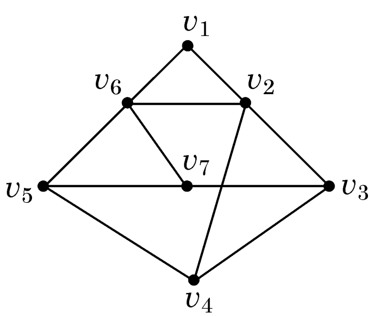
(ii) Q_n is Eulerian for all $n \geq 2$.

(iii) If G is a triangle-free graph, then $\chi(G) \leq 2$.

(iv) $K_{3,4}$ is non-planar.

(v) If G has a cut-vertex, then so does L(G).

2. (a) Use Whitney's Theorem or Expansion Lemma to show that the following graph is 2-connected : 3

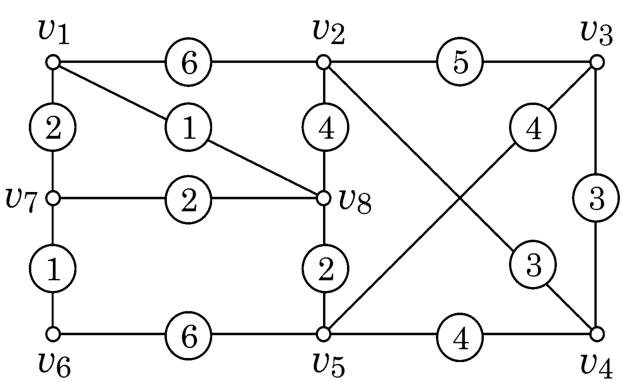


(b) Check whether the graph given in part (a) is Hamiltonian. 2

(c) Find the number of (v_3, v_5)-walks of length 3 in the graph given in part (a) by computing the powers of the adjacency matrix. 5

3. (a) Let G = (V, E) be a connected graph with at least two vertices and degree sequence $(d_1, d_2, \ldots\ldots, d_n)$, $d_i \geq 1$ for each i. Then show that G is a tree if and only if $\sum_{i=1}^{n} d_i = 2(n-1)$, where, $n = |V|$. 4

(b) Use Prim's algorithm on the following graph to find a minimum-weight spanning tree starting at v_1: 4



(c) What is the thickness of the Peterson graph ? Justify your answer. 2

4. (a) If G is a bipartite graph, then prove that $\chi'(G) = \Delta(G)$. 7

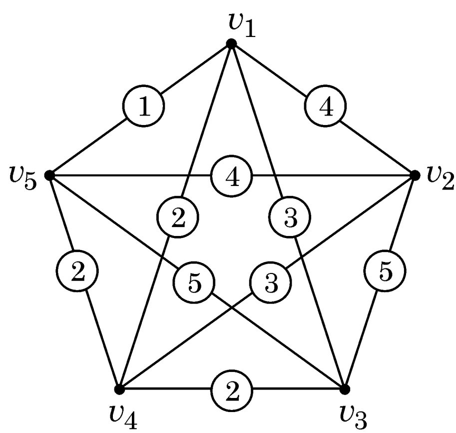
(b) Show that every proper subgraph of K_5 is planar. 3

5. (a) Show that every k-regular bipartite graph has a perfect matching, where $k \geq 1$. 5

(b) Let G be a connected graph and $v \in V(G)$. Show that ν is a cut-vertex of G iff there exist vertices $u, w \in V(G)/\{v\}$ such that every (u, w)–path in G passes through v. 5

6. (a) Start with the cycle C = $(v_1, v_2, v_3, v_5, v_4, v_1)$ in the following weighted complete

graph K_5, and perform one reduction step to get a Hamiltonian cycle with smaller weight : 4



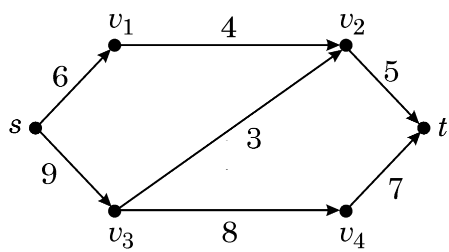
(b) Show that $K_{3,3}$ is a contraction of the Grötzsch graph. Hence conclude that Grötzsch graph is non-planar. 6

7. (a) Determine the minimum size of a maximal matching in C_n. Also determine the size of a maximum matching in C_n. Are the two quantities equal ? 3

(b) The blocks of a tree are precisely its edges. (True or false)

Justify. 2

(c) Define a flow on the following network with maximum possible value : 5



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