No. of Printed Pages : 7 **MMTE–005**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2025

MMTE-005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) There are six questions in this paper.*

*(ii) The **sixth** question is compulsory.*

*(iii) Do any **four** questions from question nos. **1** to **5**.*

(iv) Show all the relevant steps. Do the rough work at the bottom or at the side of the page only.

*(v) Calculators are **not** allowed.*

1. (a) Define the minimum distance of a code. What is the minimum distance of the following binary linear code ? 3

$$\mathbf{C} = \{0000, 1101, 1000, 0101, 1011, 0110, 0011, 1110\}$$

(b) Define the generator matrix of a linear code. Check whether : 4

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

is a generator matrix of the [4, 3] binary code :

{0000, 1101, 1000, 0101,
1011, 0110, 0011, 1110}

(c) Check that the codes $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ with generator matrices :

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

and

$$G_2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

respectively, are permutation equivalent. Find the a permutation matrix P such that $G_1P = G_2$. 3

2. (a) Define the dual of a code. Check whether the linear code $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ with generator matrices :

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $$G_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

are duals of each other. 3

(b) Let $n \in \mathbf{N}$, q be a power of a prime and $0 \leq s < n$. Define the q-cyclotomic coset of s modulo n. Find the 3-cyclotomic set of 1 modulo 23. 3

(c) Let $\boldsymbol{c}$ be a [7, 4] binary cyclic code with generator polynomial $x^3 + x^2 + 1$. Find the generator matrix and the parity check matrix of the code. 4

3. (a) Construct the Tanner graph for the given parity check matrix H of an LDPC code : 3

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(b) Find the weight distribution of the binary linear code with generator matrix :

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Use MacWilliams equations to find the weight distribution of the dual code : 7

i	α^i
1	α
2	α^2
3	$\alpha + 2$
4	$\alpha^2 + 2\alpha$
5	$2\alpha^2 + \alpha + 2$
6	$\alpha^2 + \alpha + 1$

7	$\alpha^2+2\alpha+2$
8	$2\alpha^2+2$
9	$\alpha+1$
10	$\alpha^2+\alpha$
11	$\alpha^2+\alpha+2$
12	α^2+2
13	2
14	2α
15	$2\alpha^2$
16	$2\alpha+1$
17	$2\alpha^2+\alpha$
18	$\alpha^2+2\alpha+1$
19	$2\alpha^2+2\alpha+2$
20	$2\alpha^2+\alpha+1$
21	α^2+1
22	$2\alpha+2$
23	$2\alpha^2+2a$
24	$2\alpha^2+2a+1$
25	$2\alpha^2+i$

Table 1 : Powers of $\alpha \in \mathbf{F}_{27}$, where :

$$\alpha^3+2\alpha+1=0$$

4. (a) Let $\boldsymbol{c}_1$ be the [4, 3] binary linear code generated by :

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

and let $\boldsymbol{c}_2$ be the [4, 1]-binary linear code generated by [1001]. Let $\boldsymbol{c}$ be the code obtained through using $(\mathbf{u} \mid \mathbf{u} + \mathbf{v})$ construction on the codes $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$. Find the generator matrix of $\boldsymbol{c}$. What are length and dimensions of the code ? 2

(b) Find the g.c.d. of $x^5 - x^4 + x + 1$ and $x^3 + x$ in $\mathbf{F}_5$. 3

(c) Show that the $\mathbf{Z}_4$-linear codes with generation matrices : 5

$$G_1 = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

and

$$G_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

are monomially equivalent.

5. (a) Construct a [13, 10] BCH code over $\mathbf{F}_3$ with designed distance 2. Use $x^3 + 2x + 1 \in \mathbf{F}_3[x]$ as the primitive polynomial and Table 1. 5

(b) Draw the Trellis diagram, for six cycles, for the convolutional code with generator matrix : 5

$$G = [1 + D \;\; 1 + D^2]$$

6. Which of the following statements are true and which are false ? Justify your answer with short proof or a counter-example : 2×5=10

(a) The code over $\mathbf{F}_3$ with generator matrix :

$$G = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 \end{bmatrix}$$

is self-orthogonal.

(b) Every cyclic code is self dual.

(c) $x^3 + x - 1$ is irreducible over $\mathbf{F}_5$.

(d) There are two different codes with the same generator matrix.

(e) The number of 3-cyclotomic cosets modulo 26 is 3.

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