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MPH–001

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

June, 2025

MPH-001 : MATHEMATICAL METHODS IN PHYSICS

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any **five** questions.*

(ii) The marks are indicated against each question.

(iii) Symbols have their usual meanings.

(iv) You may use calculator.

1. Using the generating function for Legendre polynomials and their orthogonality property, show that : 10

$$\int_{-1}^{+1}\left[P_n(x)\right]^2 dx = \frac{2}{2n+1}$$

2. (a) Write the Laplace's equation in spherical coordinates (r, θ, ϕ). Solve the following ODE : 2+8

$$r^2\frac{d^2R}{dr^2} + 2r\frac{dR}{dr} - n^2R = 0$$

3. (a) When is a set of vectors called linearly independent ? Show that the vectors :

$$\vec{u} = \begin{bmatrix}1\\0\\1\end{bmatrix} \text{ and } \vec{v} = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

in the Cartesian space are linearly independent. 2+3

(b) For the Pauli matrix : 5

$$\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

calculate :

$$u_1(\theta) = \exp(i\theta\sigma_2) = 1 + i\theta\sigma_2 - \frac{\theta^2}{2!}\sigma_2^2 - \ldots\ldots\ldots$$

For real θ, show that :

$$u_1(\theta) = \cos\theta_1 + i\sin\theta\,\sigma_2$$

4. (a) (i) Show that $a_i = g_{ij}\,\nu^j$ transforms covariantly, where g_{ij} are the components of the metric tensor of rank 2 and v^i the components of a contravariant vector. 3

(ii) Show that the determinant of an orthogonal matrix is either +1 or –1. 2

(b) Show that :

$$u = 2x(1-y)$$

is harmonic in some domain. Calculate its harmonic conjugate V. 2+3

5. (a) Using the method of residues, evaluate the integral : 5

$$\int_0^{\pi} \frac{d\theta}{1+\sin^2\theta}$$

(b) Obtain the Taylor's series expansion of : 5

$$f(z) = \frac{1}{z-1} \text{ about } z = -1 \text{ and } z = i.$$

6. (a) Prove that the series $\sum_{n=1}^{\infty} \frac{z^{n-1}}{3^n}$ converges for $|z| < 3$. 3

(b) Using the Laplace transforms, solve the initial value problem : 7

$$y'' - 4y' + 3y = 0;$$

$$y(0) = 3,$$

$$y'(0) = 7.$$

7. Determine the Fourier transform of the normalised Gaussian distribution :

$$f(t) = \frac{1}{\sqrt{2\pi}} \frac{1}{\tau} \exp\left(-\frac{t^2}{2\tau^2}\right) - \infty < t < \infty$$

Also draw the diagrams of the Fourier transform of Gaussian distribution for small τ and large τ. 10

8. (a) Define a continuous group giving an example. 5

(b) Show that $n \times n$ unitary matrices form a group. 5

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