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MPH–006

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

June, 2025

MPH-006 : CLASSICAL MECHANICS–II

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Attempt any **five** questions.*

(ii) The marks for each question are indicated against it.

(iii) Symbols have their usual meanings.

(iv) You may use a calculator.

1. Consider a particle of mass m with the Lagrangian :

$$L(x, y, z) = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(|\vec{r} - \vec{\mu} t|)$$

Show that a transformation $\vec{r'} = \vec{r} + \varepsilon \vec{\mu}$ leaves the Lagrangian invariant. Here ε is an infinitesimal parameter. 10

2. State Liouville's theorem. Assuming that a system in two-dimensional phase space obeys Hamiltonian dynamics, show that the volume in the phase space remains constant as the phase points evolve with time. 10

3. (a) Using the conditions $\delta_{q_i} = \delta_{p_i} = 0$ at the end points, show that : 5

$$\delta S' = \delta \int_{t_1}^{t_2} \left[p_i \dot{q}_i - H(q_i; p_i, t) + \frac{dF(q_i, p_i, t)}{dt} \right] dt = 0$$

is equivalent to the condition :

$$\delta S = \delta \int_{t_1}^{t_2} \left[p_i \dot{q}_i - H(q_i, p_i, t) \right] dt = 0$$

(b) Consider an Atwood's machine consisting of two masses m_1 and m_2, connected by a string of constant length l_0. The Lagrangian of the system is :

$$L = \frac{1}{2}(m_1 + m_2)\dot{y}_1^2 + (m_1 - m_2)gy_1 + m_2 g l_0$$

Obtain the Hamiltonian and Hamilton's equation of motion. 3+2

4. (a) Write the condition for extended canonical transformation in symplectic form. 1

(b) Using symplectic approach, show whether the transformation : 7

$$Q = \alpha^2 q \text{ and } P = \beta^2 p$$

is canonical. Here α and β are non-zero constants.

(c) Using Poisson brackets, obtain the relation for α and β, and hence the value of the scale parameter. 2

5. (a) Write the bilinear invariant condition for canonical transformation. 2

(b) Consider a dynamical system described by the Hamiltonian : 4+4

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$$

where ω is some constant.

(i) Obtain the new Hamiltonian K (Q, P), using the generating function $F_1(q, Q) = -\frac{Q}{q}$.

(ii) Obtain Hamilton's equations of motion in the new variables.

6. A particle of mass m, charge e in an EM field is described by the Lagrangian :

$$L(\vec{r}, \dot{\vec{r}}) = \frac{1}{2} m \dot{\vec{r}}^2 + e(\dot{\vec{r}} \cdot \vec{A}) - e\phi$$

where ϕ and $\vec{A}$ are the scalar and the vector potentials.

(i) Determine the canonical momenta. 2

(ii) Obtain the Hamiltonian for the system. 4

(iii) Show that : 4

$$[\vec{r}, H] = \dot{\vec{r}}$$

7. The Hamiltonian of a particle with mass m moving in a central force field is : 8+2

$$H = \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) - \frac{k}{r}$$

where k is a constant.

Obtain Hamilton's characteristic function W (r, θ) and the action-angle variables J_θ.

8. (a) Starting from $\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \vec{N}$, derive the Euler's equations for a rigid body in body axis for $\vec{L} = \omega_1 \hat{x} + \omega_2 \hat{y} + \omega_3 \hat{z}$. Here $\vec{L}$ is the angular momentum, $\vec{N}$ is the torque, and $\vec{\omega}$ is the angular velocity. 7

(b) Write Euler's equations for torque free motion. 3

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