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MASTER'S DEGREE PROGRAMME IN PHYSICS (MSCPH)

Term-End Examination

June, 2025

MPH-008 : QUANTUM MECHANICS—II

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any* ***five*** *questions.*

(ii) Symbols have their usual meanings.

(iii) You may use a calculator.

(iv) The marks for each question are indicated against it.

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1. (a) Write the rotation operator for an infinitesimal rotation ϕ_x about the x-axis. Show that the rotation operators for finite rotations are unitary. 5

(b) Define a parity transformation. Show that for the parity operator $\hat{\pi}$: 5

$$\hat{\pi}|x\rangle = |-x\rangle$$

2. Consider two non-interacting particles confined to a one-dimensional box of length a. Write the eigen functions and calculate the eigen energies of the ground state and first excited state of the system if the particles are :

(i) distinguishable

(ii) identical bosons.

Note that for a particle of mass m in a one-dimensional box of length a, the eigen functions and corresponding eigen energies are : 5+5

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}; \; E_n = \frac{\hbar^2 \pi^2 h^2}{2ma^2}$$

3. Consider the total angular momentum operator $\hat{J}$ for the addition of two angular momenta represented by the operators $\hat{J}_1$ and $\hat{J}_2$. Show that : 4+2+4

(i) $\left[\hat{J}^2, \hat{J}_1^2\right] = 0$

(ii) $\left[\hat{J}_x, \hat{J}_y\right] = i\hbar\hat{J}_z$

(iii) $\left[\hat{J}^2, \hat{J}_{1z}\right] \neq 0$

4. The ground state eigen function for a particle in a one-dimensional infinite potential well defined by the potential function :

$$V(x) = \begin{cases} 0, & \text{for } -a \le x \le a \\ \infty, & \text{elsewhere} \end{cases}$$

is $\psi_1(x) = \dfrac{1}{\sqrt{a}} \cos\left(\dfrac{\pi x}{2a}\right)$.

Calculate the first order correction to the energy for a perturbation :

$$H_1(x) = u_0 \cos\left(\frac{\pi x}{2a}\right) \text{ for } -a \le x \le a$$

where u_0 is a constant. 10

5. Determine the upper bound to the ground state energy for a potential function $V(x) = -u\delta(x)$ using a trial wave function $\psi(x) = N \exp\left(-\alpha x^2\right)$. Use $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\dfrac{\pi}{a}}$.

10

6. (a) Explain the adiabatic and sudden approximations in the context of a time-dependent Hamiltonian in quantum mechanics. 5

(b) A particle of mass m is in the ground state of a simple harmonic oscillator with a spring constant $k = m\omega^2$. At $t = 0$, the spring constant changes suddenly to $k' = 4k$. Calculate the probability that the oscillator stays in the ground state, using the sudden approximation. Before the change in the spring constant, the ground state wave function is :

$$\psi_0(x) = \left(\frac{a}{\sqrt{\pi}}\right)^{\frac{1}{2}} \exp\left(-\frac{a^2x^2}{2}\right)$$

where $a^2 = \dfrac{m\omega}{\hbar}$. You may use the following integral : 5

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}.$$

7. Write the Klein-Gordon equation. Show that $\psi(x,t) = \mathrm{N}e^{i(kx-\omega t)}$ is a solution to the Klein-Gordon equation is $\omega = \sqrt{k^2c^2 + \left(m^2c^4/\hbar^2\right)}$.

Does the conventional probability interpretation of the wave function work with the Klein-Gordon equation ? Explain. 2+5+3

8. Describe the first Born approximation. For the potential $\mathrm{V}\left(\vec{r}\right) = \Sigma_i \mathrm{U}a^3\delta\left(\vec{r} - \vec{r}_i\right)$, where U is constant and $\vec{r}_i$ are the position vectors of the vertices of a cube centred at the origin. Calculate the total scattering cross-section in the low energy limit. 3+7

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