

**POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)**

Term-End Examination

June, 2025

MST-003 : PROBABILITY THEORY

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining Question Nos. 2 to 7.*

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answers : 5×2=10

- (a) If Ω is the sample space of a random experiment, then there exists an event E such that $P(E) > P(\Omega)$.
- (b) Classical approach to probability theory works well even in the case when the outcomes of the experiment are not equally likely.
- (c) Let Ω be the sample space of a random experiment, then random variable is a function from N to Ω , where N denotes the set of all natural numbers.
- (d) If an experiment is conducted on 20 subjects whose outcomes are either success or failure only. Suppose probability of success is not constant, then it is a perfect situation for applying binomial probability distribution to obtain probability of 5 success.

- (e) If X is a continuous random variable and b is a real number, then $P(X \leq b) > P(X < b)$.
2. (a) The menstrual cycle in woman following normal distribution has a mean of 28 days and S.D. of 2 days. How frequently would you expect a menstrual cycle of (i) more than 30 days and (ii) less than 22 days ? 4
- (b) The pulse rate of healthy males follows a normal distribution with a mean of 72 per minute and S.D. of 3.5 per minute. 6
- (i) In what percentage of individuals pulse rate will differ by 2 beats from the mean ?
- (ii) Mark out symmetrical region around the mean the range in which 50% of the individuals will lie.
3. (a) Metro trains are scheduled every 5 minutes at a certain station. A person

comes to the station at a random time. Let the random variable X represents the waiting time for the next train and follows $U(0, 5)$. Find the probability that he has to wait at least 3 minutes for the train. 4

(b) State and prove memoryless property of exponential distribution. 4

(c) Write any *two* properties of Gamma distribution. 2

4. (a) In a study of drug-induced anaphylaxis among patients taking rocuronium bromide as part of their anesthesia, a doctor found that the occurrence of anaphylaxis followed a Poisson model with $\lambda = 12$ incidents per year in a certain hospital. 6

(i) Find the probability that in the next year, among patients receiving rocuronium, exactly three will experience anaphylaxis.

- (ii) What is the probability that at least three patients in the next year will experience anaphylaxis if rocuronium is administered with anesthesia ?

Take $e^{-12} = 6.144 \times 10^{-6}$.

- (b) Suppose a discrete random variable X has the following p.m.f. : 4

$$p_{(X)}(1) = \frac{1}{2}, \quad p_{(X)}(2) = \frac{1}{4}, \quad p_{(X)}(3) = \frac{1}{8},$$
$$p_{(X)}(4) = \frac{1}{8}$$

Find and sketch the c.d.f. $F_X(x)$ of the random variable X . Also find $P(X \leq 1)$ and $P(1 < X \leq 3)$.

5. Let Ω be the sample space of the random experiment of throwing one red and one black dice simultaneously. Consider the following three random variables :

X : Sum of the numbers on red and black dice.

Y : Minimum of the numbers on red and black dice.

Z : Maximum of the numbers on red and black dice.

Answer the following questions :

- (i) What are domains of X, Y and Z ? 1
 - (ii) What are ranges of X, Y and Z ? 3
 - (iii) What are probability distributions of these random variables ? 6
6. (a) If E and F are independent events, then prove that E and F are also independent. 2
- (b) Box I contains 6 red and 4 white balls. Box II contains 5 red and 3 white balls. A ball is drawn from Box I and without noticing its colour transferred to Box II. Then a ball is drawn from Box II, what is the probability that it is a red ball ? 4
- (c) Probability of solving a randomly selected problem from a book by three students X, Y and Z are 40%, 70% and 80% respectively. If a randomly selected problem is given to these three students, then what is the probability that problem will be solved ? 4

7. (a) If n randomly selected persons are in a room, then what is the probability of sharing a birthday ? Assume that each of them has his/her birthday in non-leap year. 2
- (b) Define conditional probability with example. 2
- (c) Define random variable, discrete random variable and continuous random variable, with suitable examples. 6

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