No. of Printed Pages : 4 **MST–012**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2025

MST-012 : PROBABILITY AND PROBABILITY DISTRIBUTIONS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions from question numbers **2** to **6**.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. State whether the following statements are TRUE or FALSE. Give reason in support of your answer : 5×2=10

(a) "Let (Ω, F, P) be a probability space and X be a discrete random variable which assumes countably infinite values, then we can define uniform probability law on X."

(b) If Ω = {HH, HT, TH, TT}, then write largest σ-field on Ω.

(c) Describe the Binomial distribution.

(d) Define gamma distribution with two parameters.

(e) Let X be a random variable with E(X) = 3, then which one of the following is true and why ?

$$E\left(\frac{1}{X+2}\right) \geq \frac{1}{5} \text{ or } E\left(\frac{1}{X+2}\right) < \frac{1}{5} \; \forall x > -2$$

2. State Monty Hall problem and solve it. Also, write its intuitive solution. 10

3. (a) A bag contains 4 red and 12 black balls. Three balls are drawn one-by-one with replacement. Plot PMF and CDF of number of red balls. 6

(b) Let (Ω, F, P) be a probability space and E be an event, then find the probability distribution of the indicator random variable $I_E : \Omega \to \{0, 1\}$ of event E. 4

4. (a) Suppose a biased coin having p as probability of success is given to you, where p is unknown. Initially, to give full freedom to the value of p, you are considering p as a random variable, where $p \sim$ unif (0, 1). You tossed the coin 100 times and observed 98 times head and 2 times tail. With this information, find the probability that p is greater than 0.8. 5

(b) If each of the random variables X, Y and Z follow exponential distribution with parameter 2, then find $P(X + Y + Z \leq 1)$. 5

5. State and prove Borel-Cantelli lemmas 1 and 2. 10

6. The PDF of jointly continuous random variable (X and Y) is given by :

$$f_{X,Y}(x, y) = \begin{cases} k(4-x)(5-y), & 0 \le x \le 4,\ 0 \le y \le 5 \\ 0 & , \text{ otherwise} \end{cases}$$

Obtain : 1+1+5+1+1+1

(i) The value of k

(ii) Marginal PDFs of X and Y

(iii) Joint CDF of (X, Y)

(iv) Marginal CDFs of X and Y

(v) Conditional PDF of X given Y

(vi) Are random variable X and Y independent ?

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